Performance

6. Airfoils and Wings

The primary lifting surface of an aircraft is its wing. The wing has a finite length called its wing span. If the wing is sliced with a plane parallel to the x-z plane of the aircraft, the intersection of the wing surfaces with that plane is called an airfoil. This airfoil shape can be different if the slice is taken at different locations on the wing. However, for any given slice, we have a given airfoil. We can now think of the airfoil as an infinitely long wing that has the same cross sectional shape. Such a wing (airfoil) is called a two dimensional (2-D) wing. Therefor, when we refer to an airfoil, you can think of an infinite wing with the same cross sectional shape. Since calculating lift and drag coefficients with a reference area of infinity, would not make sense, we base airfoil lift and drag coefficients for airfoils on the planform area, assuming the span is unity.

Airfoil Geometry and Nomenclature (2-D)

The figure at the right is a 2-D airfoil section. It consists of the leading edge (LE), the trailing edge (TE) and the line joining the two called the chord (c). The angle-of-attack is generally measured between the velocity (or relative velocity) vector V and the chord line.(Although the angle-of-attack can be defined as the angle between the velocity vector and any fixed line in the airfoil).
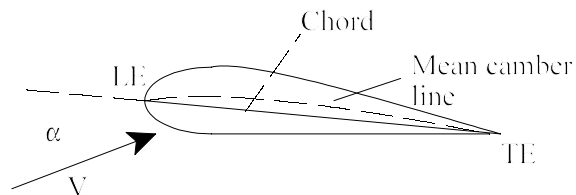
A line that is midway between the upper surface and lower surface is called the camber line. The maximum distance from the chord line to the camber line is designated as the airfoil camber (δ), generally expressed as a percent of the chord line, such as 5% camber. The maximum distance between the upper and lower surface is the airfoil thickness, t_{max}, also designate as a percent of chord length. The we have:

$$\frac{\delta}{c} \cdot 100 \; = \; \% \, \text{camber} \qquad \frac{t_{max}}{c} \cdot 100 \; = \; \% \, \text{thickness} \qquad (1)$$

As defined earlier, the lift and drag on an airfoil are defined perpendicular and parallel to the relative wind respectively. In addition, we can define the aerodynamic pitch-moment relative to some point on the airfoil (usually located on the chord), with the sign convention that a positive pitch moment is in the direction that would move the nose up. (If we recall, that the y body axis points out the right hand wing, then the moment about the y axis, using the right hand rule, would give us a nose up moment as positive).

We generally designate airfoil 2-D aerodynamic properties by lower case letters. For

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example the lift coefficient, 2-D is C_l as compared to C_L used for the 3-D lift coefficient. With this in mind, we can define the 2-D lift, drag, and pitch moment in the following manner:

$$C_l = \frac{L}{1/2\,\rho\,V^2\,c\,(1)}$$

$$C_d = \frac{D}{1/2\,\rho\,V^2\,c\,(1)} \qquad\qquad f(\alpha,Re,M) \qquad (2)$$

$$C_m = \frac{M}{1/2\,\rho\,V^2\,c^2\,(1)}$$

where c (1) is the chord times the unit width that we use for area in the case of 2-D bodies. We can also note that the pitch-moment requires an additional length in the denominator to retain a non-dimensional form; here we use the chord length.

The National Advisory Committee for Aeronautics (NACA) did systematic tests on various shaped airfoils in order to generate a data base for aircraft design. Although performed a long time ago, these data are still used when designing certain appendages of the aircraft. The system consists of a series of 4, 5 and 6 digit airfoils.

4-digit airfoils (e.g. NACA 2415):

      2 - maximum camber is 0.02% over the chord, $\delta = 0.02\,c$
      4 - the location of the maximum camber along the chord line given as $0.4\,c$
      15 - the maximum thickness, here $0.15\,c$

5-digit airfoils (e.g. NACA 23021):

      2 - maximum camber is 0.02% over the chord, $\delta = 0.02\,c$
      30 - the location of the maximum camber along the chord line /2, here, $0.15\,c$
      21 - the maximum thickness, here $0.21\,c$
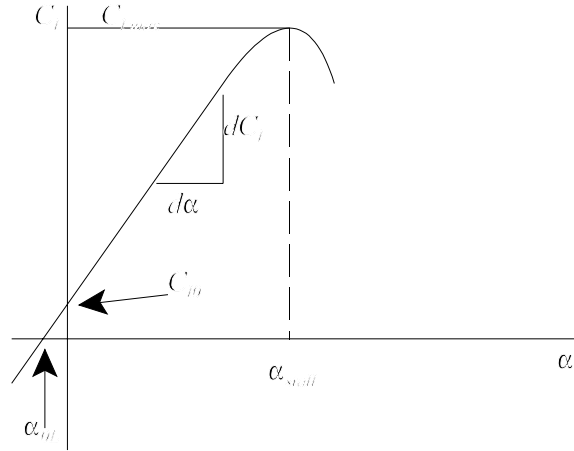
6-digit airfoils (e.g. NACA 63₂215):

      6 - series designator
      3 - maximum pressure location, here, $0.3\,c$
      $_2$ - minimum drag at design lift coefficient, $\pm\,0.2$
      2 - design lift coefficient, here, $C_{l_{design}} = 0.2$
      15 - the maximum thickness, here $0.15\,c$

Typically, tail surfaces of an aircraft are symmetric and are made with thin airfoils such as an NACA 0012. (Zero camber, 12% thick).

Aerodynamic Properties (2-D)

<u>*Lift Characteristics*</u>

The aerodynamic properties of most interest to us for performance considerations are those associated with lift and drag. A plot of lift coefficient vs angle-of-attack is called the lift-curve. A typical lift curve appears below. We can note the following: 1) for small angles-of-attack, the lift curve is approximately a straight line. We will make that assumption and hence deal almost exclusively with "linear" aerodynamics. 2) That for some angle-of-attack called the **stall angle-of-attack**, the lift coefficient reaches a maximum, $C_{l_{max}}$. 3) There are two intercepts that we can designate, one the alpha axis for zero lift, designated as α_{0L}, the zero-lift angle-of-attack, and the one at zero angle-of-attack designated at C_{L_0} , the lift at zero angle-of-attack.

With the assumption of linear aerodynamics, we can create a mathematical model of how the lift coefficient varies with angle-of-attack. To simplify the resulting expressions, we can first define the 2-D lift-curve slope:

$$a_0 \ = \ \frac{d\,C_l}{d\,\alpha} \tag{3}$$

where the subscript "0" is used to designate that this is a 2-D lift-curve slope. With this definition, we can write our mathematical model for the lift coefficient:

2-D Lift Curve

$$
\begin{aligned}
C_l &= C_{l_0} \ + \ a_0\,\alpha \\
&= a_0\left(\alpha \ - \ \alpha_{0L}\right)
\end{aligned}
\tag{4}
$$

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where it is easily seen that $C_{l_0} = -a_0 \alpha_{0L}$ or $\alpha_{0L} = -\dfrac{C_{l_0}}{a_0}$. My preference as to form is the one that uses α_{0L}. However, both forms are used by various authors.

2 -D Moment

     In order to calculate an aerodynamic moment for an airfoil, we need to define a reference point about which to define the moment. Typical reference points are the leading edge of the airfoil and the 1/4 chord location of the airfoil (for reasons to be determined later). The force and moment system on an airfoil is shown in the figure:
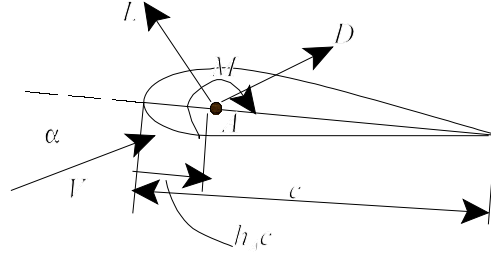
     The drag is parallel to the relative wind, and the lift is perpendicular to the relative wind. The aerodynamic moment is positive nose up. Here we are taking the moment about point A. Once we pick a point, we can use some theorems from statics that say that we can represent and force and moment system by assuming that the forces act thorough a given point and that there is a pure moment about that point.

     Here we locate the reference point from the leading edge of the airfoil at a distance, $h_A c$ from the leading edge of the airfoil along the chord line. We could also select another point, B and assume the lift and drag act through that point, and that, in addition, there is a pure moment about B (different from that about A). We can arrive at an equation that allows us to transfer moments from one point to another in the following way: Consider taking moments about the leading edge of the airfoil. Then we have:

$$M_{LE} = M_A - L_A \cos\alpha\,(h_A\,c) - D_A \sin\alpha\,(h_A\,c) = M_B - L_B \cos\alpha\,(h_B\,c) - D_B \sin\alpha\,(h_B\,c)$$

However the forces are the same so that $L_A = L_B = L$, and $D_A = D_B = D$. The moments are different and are related by the above equation that we can rewrite as:

$$M_A = M_B + L\,(h_A - h_B)\,(c)\cos\alpha + D\,(h_A - h_B)\,(c)\sin\alpha \qquad (5)$$

This equation can be simplified by making a few observations: 1) the angle α is $\ll \pi$, so that the cosine of the angle is approximately 1 and the sine of the angle equals the angle, $\cos\alpha = 1$, and $\sin\alpha = \alpha$, 2) the lift is much less than the drag, $L \ll D$. With these two assumptions, we can note that the last term in Eq. (5) is the product of two small quantities and is therefor second order compared to the first two terms, and can be neglected. With these assumptions we have the

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equation that we are looking for:

$$M_A = M_B + L(h_A - h_B)c \tag{6}$$

If we put this in coefficient form (see Eq. (2)), then we have:

General Rule for Transferring Moments

$$C_{m_A} = C_{m_B} + C_l(h_A - h_B) \tag{7}$$

Equation (7) is used for changing reference points for taking moments. Here $h_{()}$ is the non-dimensional location (in chord lengths) of the reference point from the leading edge of the wing.

Example

How is the moment about the leading edge of the wing related to the moment about point A? All be need to do is let point B be the leading edge of the airfoil, then $h_B = 0$ and we have:

$$C_{m_{LE}} = C_{m_A} + C_l(0 - h_A) = C_{m_A} - C_l h_A$$

Definition Aerodynamic Center

The aerodynamic center is the reference point about which the aerodynamic moment does not change with changes in angle-of-attack:

$$\frac{dC_{m_{ac}}}{d\alpha} \equiv 0 \tag{8}$$

The location of the aerodynamic center can be determined from experimental data from its definition:

$$C_{m_{ac}} = C_{m_B} + C_l(h_{ac} - h_B)$$

and

$$\frac{dC_{m_{ac}}}{d_\alpha} = 0 = \frac{dC_{m_B}}{d\alpha} + \frac{dC_l}{d\alpha}(h_{ac} - h_B) \quad \Rightarrow \quad h_{ac} = h_B - \frac{\dfrac{dC_{m_B}}{d\alpha}}{\dfrac{dC_l}{d\alpha}} \tag{9}$$

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Consequently, if we took some wind tunnel data a measured the lift and the moment about some reference point and found how lift and moment would change with angle-of-attack, we could determine the aerodynamic center.

Example

Wind tunnel data was taken on an airfoil and the following data taken at the 1/3 chord location:

C_l	0.2	0.4	0.6	0.8
$C_{m1/3}$	-0.02	0.0	0.02	0.04

We can note that $\dfrac{\dfrac{dC_{m_\alpha}}{d\alpha}}{\dfrac{dC_l}{d\alpha}} = \dfrac{dC_m}{dC_l}$, hence we have $h_{ac} = h_B - \dfrac{dC_{m_B}}{dC_l}$. We can calculate the slope

from the data: $\dfrac{dC_{m_{1/3}}}{dC_l} = \dfrac{0.04 - (-0.02)}{0.8 - 0.2} = 0.10$, and

$$h_{ac} = h_B - \frac{dC_{m_B}}{dC_l} = \frac{1}{3} - 0.1 = 0.233$$

Therefor the aerodynamic center is located at the 23.3% chord location.

Since the pitch moment (coefficient) is constant at the aerodynamic center, it is evident that when the lift is zero, the pitch moment at the aerodynamic center is unchanged and is still the same. However from Eq. (7), it is clear that when there is zero lift, the pitch moment is the same about any point on the airfoil (since there are no forces, it is a pure couple). Consequently, we can note that:

$$C_{m_{0L}} = C_{m_{ac}} = \text{constant} \tag{10}$$

From our example problem (extending the graph): $C_{m_{0L}} = C_{m_{ac}} = -0.04$.

Typically the aerodynamic center is very close to $h_{ac} = 0.25$ for subsonic flow and $h_{ac} = 0.5$ for supersonic flow. From Eq. (9) it can be seen for linear aerodynamics (slopes are constants) that the aerodynamic center location is also constant.

Definition - Center of Pressure

The center of pressure is defined as the location on the airfoil where the ***pitch moment is zero***. We can determine an expression for the center of pressure from the general moment

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equation, Eq. (7)

$$C_{m_{cp}} = 0 = C_{m_B} + (h_B - h_{ac}) \quad \Rightarrow \quad h_{cp} = h_B - \frac{C_{m_B}}{C_l} \qquad (11)$$

It would be convenient to make the point B the aerodynamic center so that $C_{m_B} = C_{m_{ac}} = \text{const}$
Then the center of pressure would be:

$$h_{cp} = h_{ac} - \frac{C_{m_{ac}}}{C_l} \qquad (12)$$

For our sample problem, for the case where the lift coefficient is 0.2, the center of pressure would be located at

$$h_{cp} = h_{ac} - \frac{C_{m_{ac}}}{C_l} = 0.233 - \frac{-0.4}{0.2} = 0.433$$

Note that unlike the aerodynamic center, the location of the center of pressure depends upon the lift coefficient.

Airfoil Drag Characteristics

The drag on an airfoil (2-D wing) is primarily due to viscous effects at low speed and compressibility effects (wave drag) at high speed. In addition, at high angles of attack, the flow can separate from the upper surface and cause additional drag. Hence as indicated in our dimensional analysis, the drag coefficient depends on three quantities, Reynolds number, Mach number, and the angle-of-attack. Typically the Reynolds number is important at low speeds, the Mach number at high speeds and the angle-of-attack at all speeds. Some typical curves are shown below:
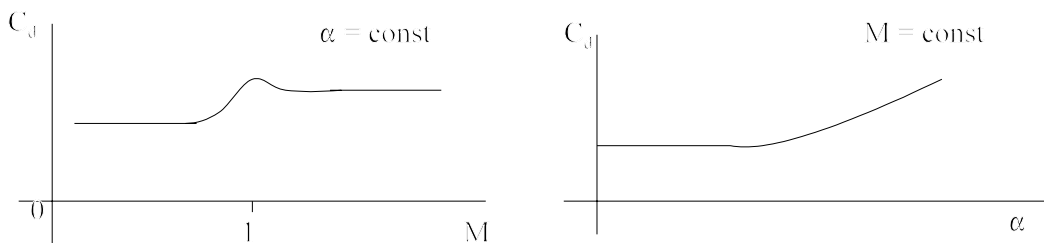

Here we see that the drag coefficient is nearly constant at subsonic speeds and tends to rise just before Mach = 1. The biggest variation is in the neighborhood of Mach 1, called the transonic region. Above that region, say about Mach 1.2, the 2-D drag coefficient tends to be constant or it could increase or decrease slightly. The figure on the right represents a typical change of drag coefficient with angle-of-attack at a given Mach number. It tends to increase slightly with angle-

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of-attack at low angles, and increases more rapidly at high angles-of-attack. The curve is approximately quadratic in angle-of-attack.

If we look at a close up of the drag coefficient in the transonic region, we can define certain specific Mach numbers.

As the Mach number is increased, the first specific Mach number that we encounter is the *critical Mach number*, M_c.
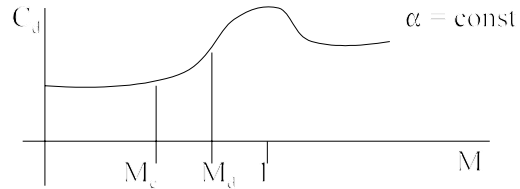
Definition: Critical Mach Number
The critical Mach number is defined as the Mach number at which the flow somewhere on the airfoil is sonic, Mach = 1.

The next Mach number encountered is called the ***Drag Divergence Mach Number***.

Definition: Drag Divergence Mach Number
The drag divergence Mach number is that Mach number where the drag coefficient increases by 0.002 or by 2 "counts" of drag. Note that this definition is not universal. Other definitions exist, some based on the slope rather than the value itself.

Finite (3-D) Wings
If we now take an airfoil and make it finite in length we have a lifting surface called a wing. Although wings can be in many shapes and sizes, here we will limit our discussion and definitions to straight tapered wings. The planform view of a straight tapered wing is shown in the figure:
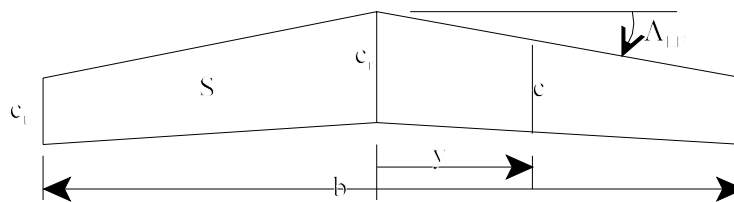
The following geometric properties can be defined:

S	=	wing area (planform)
b	=	wing span
c_t	=	tip chord
c_r	=	root chord

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λ = c_t/c_r = taper ratio
Λ_{LE} =   leading edge sweep angle
Λ_n =   Sweep angle at the n chord location $0 \le n \le 1$ (0 leading edge, 1 trailing edge)

Wc can also define several different chords that characterize the wing.

Definition: Mean Geometric Chord, c_g

The mean geometric chord is the chord of a rectangular wing having the same span and the same area as the original wing. It can be found for any general wing in the following way:

$$c_g = \frac{\displaystyle\int_0^{b/2} c(y)\, dy}{\displaystyle\int_0^{b/2} dy} = \frac{2}{b}\int_0^{b/2} c(y)\, dy = \frac{S}{b} \tag{13}$$

For a straight tapered wing:

$$c_g = \frac{c_r}{2}(1 + \lambda) \tag{14}$$

Definition: Mean Aerodynamic Chord, $\bar{c}$

The mean aerodynamic chord is (loosely) the chord of a rectangular wing with the span, (not area) that has the same aerodynamic properties with regarding the pitch-moment characteristics as the original wing. It can be found for any general wing in the following way:

$$\bar{c} = \frac{\displaystyle\int_0^{b/2} [c(y)]^2\, dy}{\displaystyle\int_0^{b/2} c(y)\, dy} = \frac{2}{S}\int_0^{b/2} [c(y)]^2\, dy \tag{15}$$

For a straight tapered wing this equation gives:

$$\bar{c} = \frac{2}{3} c_r \frac{1 + \lambda + \lambda^2}{1 + \lambda} \tag{16}$$

To characterize the general "shape" of the wing, we can define the aspect ratioL

Definition; Aspect Ratio, AR

      The aspect ratio is the wing span divided by the mean geometric chord. It is a measure of how long and narrow a wing is. A square wing would have an aspect ratio of 1! We can calculate the aspect ratio in several ways:

$$AR = \frac{b}{c_g} = \frac{b^2}{S} \tag{17}$$

Definition: Wing Sweep Angle

      The wing sweep angle is the angle between a perpendicular to the centerline and the leading edge of the wing. We can also define it as the angle to a line drawn through the nth cord location of every wing chord. For example, the quarter chord sweep angle. If we know the sweep angle at one chord location, we can find it an any other sweep location from:

$$\tan \Lambda_n = Tan\, \Lambda_m - \frac{4}{AR}\left[(n-m)\frac{1-\lambda}{1+\lambda}\right]$$

Wing Aerodynamic Properties (3-D)

      The fact that a wing is of finite length has considerable effect on its aerodynamic properties. The primary effect is due to the span-wise lift distribution (it is no longer constant), caused by the flow about the wing tips. In normal operating conditions, the wing will have high pressure on it lower surface and a low pressure on its upper surface. This pressure difference is what generates the lift. However, this same pressure difference causes flow from the under side of the wing to the upper side of the wing around the wing tips.
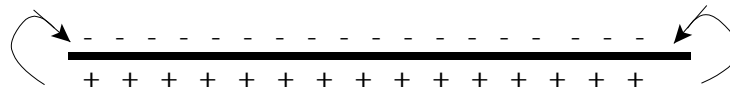
This type of flow swirls off the tips of the wing in the form of vortices. In fact there is a vortex distribution across the entire span of the wing with the strongest vortices at the wing tips. These vortices trail downstream behind the wing and rotate in the direction shown in the figure. Vortices on the right hand side of the wing (looking from the rear) rotate counter clockwise, and those on the left hand side of the wing rotate clockwise. The general result is that the vortices induce a downward flow at the wing interior. This downward flow is called downwash, and it influences the flow in front of, at, and behind the wing. This downward flow causes a change in the local wing angle-of-attack such that the wing sees a different angle-of-attack then the one that it sees with respect to the free stream.

The velocity induced by the vortices is designated as the downwash, w. This downwash decreases the local angle-of-attack by an amount called the induced angle-of-attack, α_i that is approximately (small angle approximation) given by

$$\alpha_i = \frac{w}{V} \qquad (18)$$
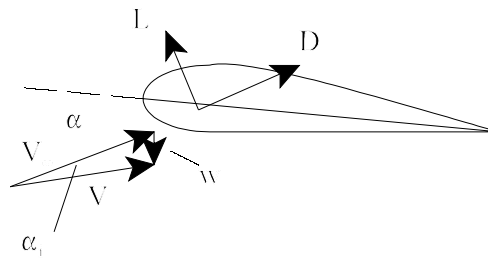
As a result, the local *airfoil* only "sees" the "effective" angle-of-attack(or what we called in the previous section the 2-D angle-of-attack). Hence we have:

$$\alpha_{eff} = \alpha - \alpha_i \qquad (19)$$

where:
|		
α	=	actual angle-of-attack measured with respect to the free stream
α_i	=	induced angle-of-attack
α_{eff}	=	effective angle-of-attack, the local angle of attack seen by the local airfoil section, also could be called the 2-D angle-of-attack

At this point, we can look at the local section and determine the lift from the 2-D section properties that we examined previously. In doing so, we can observe that the effect on the local airfoil section calculations is to:

1) reduce the amount of lift for a given α since

$$c_l = a_0 \left(\alpha_{eff} - \alpha_{0L} \right) < a_0 \left(\alpha - \alpha_{0L} \right)$$

E.g. the local airfoil sees a smaller angle-of-attack then the free stream angle-of-attack

2) cause the relative wind to come in at a different direction (lower angle-of-attack) and hence the lift perpendicular to it will be in a different direction from the defined lift direction (perpendicular to the free stream) thus producing a component of force in the drag direction called induced drag.
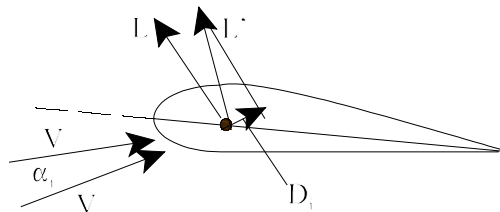
From the figure we have:

$$L = L' \cos \alpha_i \approx L'$$

$$D_i = L' \sin \alpha_i \approx L' \alpha_i$$

Therefor the finite wing causes a downwash that causes a change in the local relative wind direction so that the lift generated perpendicular to this local relative wind is no longer perpendicular to the free-stream velocity. It is tilted backward a small amount. The component of this lift parallel to the original free stream direction is called the induced drag.

We now consider estimating this induced angle-of-attack and the accompanying induced drag. We will use the momentum theory developed previously. Consider a circular tube of air whose diameter is equal to the span of the wing. The air enters and leaves the tube with the free stream velocity, V. It enters the tube, far upstream of the wing, with an angle equal to zero and leaves the tube, far downstream of the wing, deflected downward by a small amount, ϵ. This angle is known to be twice the downwash angle at the wing, $\epsilon = 2 \alpha_i$.
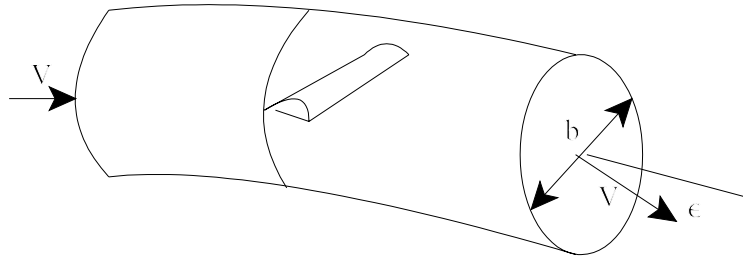
The momentum theory tells us that the force applied to the fluid must equal the change in momentum in that direction. Here we are interested in the force that the wing applies to the fluid that pushes the fluid down. The opposite of that force is the force on the wing by the fluid, or the lift. The momentum theory tells us that: (in the "z" direction)

$$F_{\text{wing on fluid}_z} = (\text{momentum flux out} - \text{momentum flux in})_z \qquad (20)$$

where the momentum flux is the mass flow rate ($\rho A V$) times the component of the velocity in the direction of interest. Since the flow in is perpendicular to the "z" direction, there is no momentum flux in, in the z direction. Hence we have:

$$F_z = \left[\rho \left(\frac{\pi \, b^2}{2} \right) V \right] V \sin \epsilon \; - \; 0$$

But the force in the z direction (positive down) is the wing force on the fluid. Therefor the force of the fluid is the negative of that force and acts upward and is called the lift (perpendicular to the free-stream velocity). Accounting for the sign changes (force on wing opposite F_z, and lift having the opposite sign of F_z) we can write an expression for lift:

$$L = C_L \frac{1}{2} \rho \, V^2 S = \rho \left(\frac{\pi \, b^2}{4} \right) V^2 \, \epsilon = \rho \left(\frac{\pi \, b^2}{4} \right) V^2 \, (2) \, \alpha_i$$

If we solve for the downwash angle at the wing, we get:

$$\alpha_i = \frac{C_L}{\pi \, AR} \tag{21}$$

The induced drag is then obtained from:

$$D_i = L \frac{C_L}{\pi \, AR} = \bar{q} \, S \frac{C_L^2}{\pi \, AR} \qquad\qquad C_{D_i} = \frac{C_L^2}{\pi \, AR} \tag{22}$$

The results in Eqs. (21, and 22) are somewhat restricted. They can be generalized in an "engineering" way, by introducing an engineering "factor" For the general case we can introduce the "*span efficiency factor*," e_w:

$$C_{D_i} = \frac{C_L^2}{\pi \, AR \, e_w} \tag{23}$$

The total drag coefficient for a wing is then given by:

$$C_D = C_d + \frac{C_L^2}{\pi \, AR \, e_w} \tag{24}$$

where:

C_d = 2-D drag coefficient due to skin friction, separation, wave drag

e_w = span efficiency factor (usually less than 1)

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$$AR \quad = \quad \text{wing aspect ratio}$$
$$C_L \quad = \quad \text{wing lift coefficient}$$

Aircraft Drag Coefficient

We can now consider the drag coefficient of the entire aircraft. One can note that there are other lifting surfaces on the aircraft such as the tail or canard surface. Each of these surface's contribution to the drag is similar to Eq.(24). Also the fuselage and other parts of the aircraft contribute terms that look similar to Eq. (24). When we combine all these terms we can arrive at a single representative drag expression for the complete aircraft. This expression looks similar to Eq. (24) and is given by:

Aircraft Drag Coefficient (Drag Polar)

$$C_D = C_{D_{0L}} + \frac{C_L^2}{\pi \, AR \, e}$$
$$= C_{D_{0L}} + K C_L^2 \tag{25}$$

where: $C_{D_{0L}}$ =   zero-lift drag coefficient, parasite drag coefficient

e   =   Oswald (aircraft) efficiency factor

K   =   induced drag parameter

Drag Polars

The drag coefficient can be written as functions of many different variables. The strongest dependence of drag is on the lift coefficient because of the induced drag effect. If the mathematical model for the drag coefficient contains the lift coefficient, that expression is called a ***drag polar***. Hence we can make the following definition:

<u>Definition: Drag Polar</u> A drag polar is a any mathematical expression the relates the drag to some function of the lift coefficient, $C_D = f(C_L)$.

There are several different drag polars that are typically used to represent the drag of an aircraft. The following two are the most frequently used when Eq. (25) is not used:

$$C_D = C_{D_{0L}} + K_1 C_L + K_2 C_L^2$$

$$C_D = C_{D_{min}} + K\left(C_L^2 - C_{L_0}^2\right)$$

(26)

where C_{L_0} is the lift coefficient when the drag is a minimum. The last equation in Eq. 26, and Eq. (25) are examples of a particular type of a drag polar, a **parabolic drag polar**. For the majority of this course we will use the **parabolic drag polar given in Eq. (25)**

In general the drag coefficient is a function of Reynolds number and Mach number. In general, for the normal flight regime, the dependence on these numbers is the similar to the dependence as observed on the airfoil previously. Typically, in the flight conditions of interest, the Reynolds number has only a small effect while that of the Mach number is prominent in high subsonic, transonic, and supersonic flight. We can indicate this dependence on Mach number in our parabolic drag polar as follows:

$$C_D = C_{D_{0L}}(M) + K(M) C_L^2$$

(27)

where $C_{D_{0L}}(M)$ $\quad = \quad$ Mach number dependent zero-lift drag

$\quad K(M)$ $\quad\quad = \quad$ Mach number dependent induced drag parameter

We could also write: $K(M) = \dfrac{1}{\pi\, AR\, e(M)}$, where the oswald efficiency factor is a function of the Mach number.

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Example

An aircraft weighs 40,000 lbs, wing area of 350 ft^2 and a wing span of 50 ft. At sea-level the aircraft flies at 200 and 600 ft/sec. What are the values of the induced drag and the associated drag coefficients for this case. Noting that Lift = weight in level flight:

$$C_{L_1} = \frac{W}{1/2\, \rho\, V_1^2\, S} = \frac{40,000}{1/2\,(0.002377)\,(200)^2\,(350)} = 2.400$$

$$C_{L_2} = \frac{W}{1/2\, \rho\, V_2^2\, S} = \frac{W}{1/2\, \rho\, V_1^2\, S} \cdot \frac{V_1^2}{V_2^2} = 2.400\left(\frac{200}{600}\right)^2 = 0.267$$

$$AR = \frac{b^2}{S} = \frac{50^2}{350} = 7.143 \qquad\qquad \text{Also assume e} = 0.85$$

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$$C_{D_{i_1}} = \frac{C_L^2}{\pi\, AR\, e} = \frac{2.400^2}{\pi\,(7.143)\,(0.85)} = 0.301$$

$$C_{D_{i_2}} = \frac{0.267^2}{\pi\,(7.143)\,(0.85)} = 0.004$$

$D_1 = C_{D_i}\, 1/2\, \rho\, V^2 S = 0.301\,(1/2)\,(0.002377)\,(200^2)\,(350) = 5014.7$ lbs Induced drag only!

$D_2 = C_{D_i}\, 1/2\, \rho\, V_2^2 S = 0.004\,(1/2)\,(0.002377)\,(200^2)\,(350) = 599.8$ lbs Induced drag only!

Wing Lift Curves - 3-D effects

As we observed previously, the finite wing sees a reduced local angle-of-attack. The local airfoil section sees the α_{eff} while the wing is at a real angle-of-attack, α, where $\alpha_{\mathit{eff}} = \alpha - \alpha_i$.

The result of this difference is that for a given angle-of-attack, the finite wing has less lift since it sees a smaller angle. The lift of the wing can be found from:

$$C_L = a_0\, \alpha_{\mathit{eff}} = a_0\,(\alpha - \alpha_i) = a_0\left(\alpha - \frac{C_L}{\pi\, AR\, e_w}\right) \tag{28}$$

We can solve for C_L to get:

$$C_L = \frac{a_0\, \alpha}{1 + \dfrac{a_0}{\pi\, AR\, e_w}} \tag{29}$$

More importantly, we can see that the finite wing modifies the lift-curve slope. The 3-D lift curve slope is determined from Eq. (29) to be:

3-D lift-curve slope (Prandtl's formula, Low speed, high aspect ratio, little sweep)

$$\frac{dC_L}{d\alpha} = a = \frac{a_0}{1 + \dfrac{a_0}{\pi\, AR\, e_w}} \tag{30}$$

Equation (30) relates the 3-D lift-curve slope to the airfoil 2-D lift-curve slope. The units of a_0 must be per radian (at least in the denominator).

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Example:

The aircraft from the previous example has a wing with a 2-D airfoil section that has a lift-curve slope of $a_0 = 2\pi$. What is the lift-curve slope of the wing?

The wing aspect ratio was calculated to be AR = 7.143. We will assume that $e_w = 0.85$. Then the wing lift-curve slope is calculated from:

$$a = \frac{a_0}{1 + \dfrac{a_0}{\pi\,AR\,e_w}} = \frac{2\pi}{1 + \dfrac{2\pi}{\pi\,(7.143)\,(0.85)}} = 4.726 \text{ /rad} = 0.082 \text{ /deg}$$

We see that $a_0 = 2\pi = 6.28$ /rad, while $a = 4.726$ /rad,

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The above equation (Eq. 30) is generally valid for low speed high aspect ratio wings. An alternate expression for the 3-D lift-curve slope for straight tapered wings performing both high or low speed subsonic flight, that includes the effects of Mach number and wing sweep is given by the following expression:

DATCOM Formula
Straight tapered wing lift-curve slope (subsonic, all speeds and AR)

$$a = \frac{2\pi\,AR}{2 + \sqrt{\dfrac{AR^2\,(1 - M^2)}{k^2}\left[1 + \dfrac{\tan^2\Lambda_{1/2}}{(1 - M^2)}\right] + 4}} \tag{31}$$

Where $k = \dfrac{a_0}{2\pi}$, and a_0 = actual 2-D lift-curve slope of airfoil section. If unknown assume $k = 1$.

For supersonic flow, low aspect ratio, straight tapered wings we have the following mathematical model for the lift-curve slope:

Straight tapered wing lift-curve slope (supersonic, low aspect ratio)

$$a = \frac{4}{\sqrt{M^2 - 1}}\left(1 - \frac{1}{2\,AR\,\sqrt{M^2 - 1}}\right) \tag{32}$$