


- NTNU
- Noregs teknisk-naturvitskapelege universitet
- Fakultet for naturvitskap og teknologi
- Institutt for fysikk



- Fag **ENERGI OG MILJØFYSIKK - TFY4300**
- **(Energy and Environmental Physics)**

## OCEAN-WAVE ENERGY

Introductory lectures by  
Johannes Falnes

14 & 15 September 2016

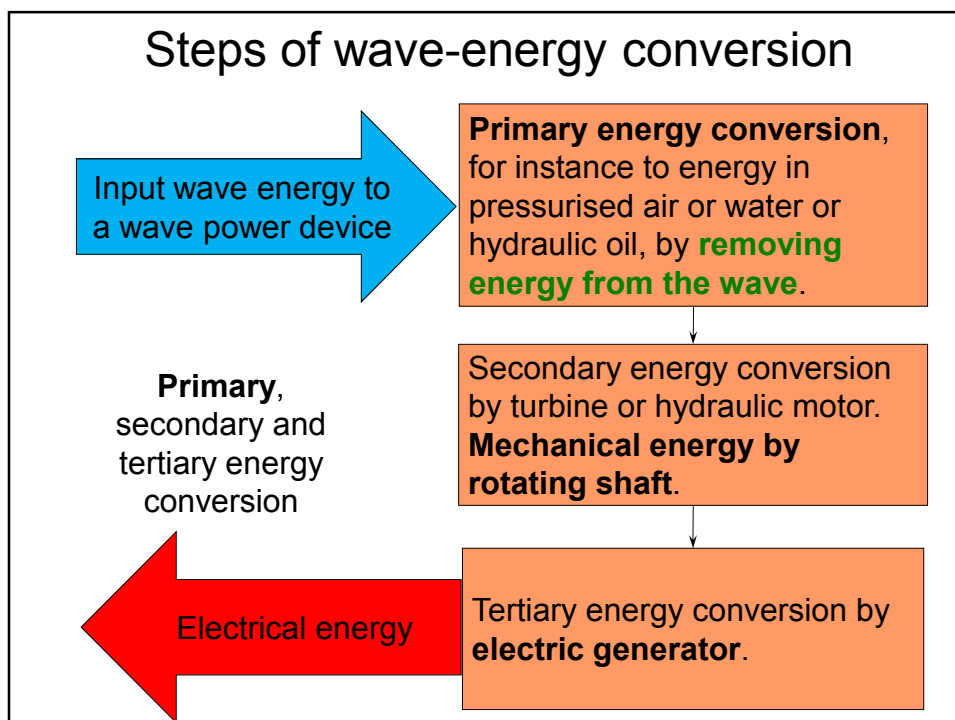
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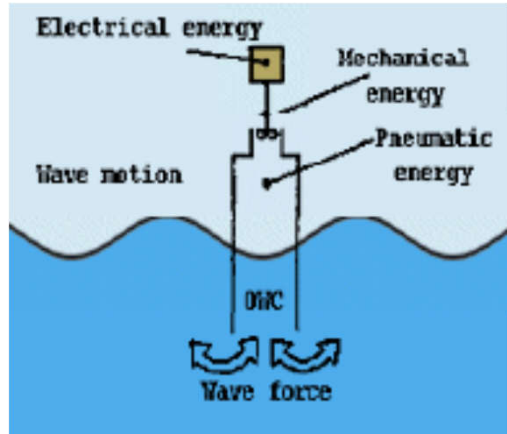
### Ocean waves as energy resource

- Ocean waves represent a clean and renewable energy source, come into being by conversion of wind energy when winds blow along the sea surface. Wind energy, in turn, originates from solar energy, because sun heating produces low pressures and high pressures in the atmosphere. In either of these two energy conversions, energy flow becomes intensified.
- Just below sea surface the average wave-power level (energy transport) is typically ten times denser than the wind energy transport 20 m above the water, and 30 to 50 times denser than average solar energy intensity.

- Average energy intensity:
  - Solar energy: 100 - 200 W/m<sup>2</sup>
  - Wind energy: 400 - 600 W/m<sup>2</sup>
  - Wave energy: 4 - 6 kW/m<sup>2</sup>  
(just below the sea surface  
- but less in deeper water)



Unfortunately, it seems that some inventors do not understand the very important step of primary energy conversion.



Is this an illustration of a wave-energy converter?

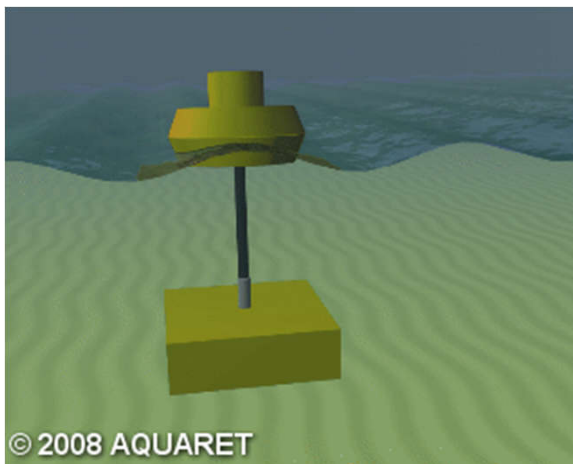
In the third step, an electric generator is to deliver electricity.

In the second step an air turbine is to deliver mechanical energy to a rotating shaft.

Nothing seems to happen with the wave! Thus the net **primary converted energy is zero!**

According to the illustration, if this device delivers useful energy, it appears to be a **perpetual engine machine!** No energy seems to be removed from the wave.

This “quasi point absorber” (QPA) seems to absorb no wave energy!

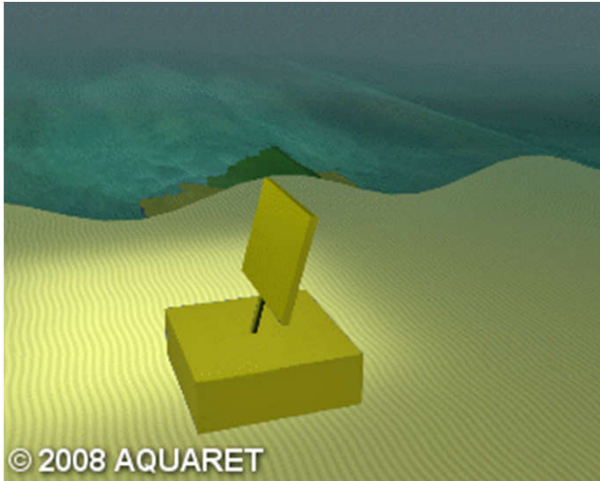


Nothing seems to happen with the wave!

If this device delivers energy, it seems to be a **perpetual engine machine!**

Animation downloaded 2016-08-26 from <http://www.emec.org.uk/marine-energy/wave-devices>

This “pitching-flap absorber” seems to absorb no wave energy!



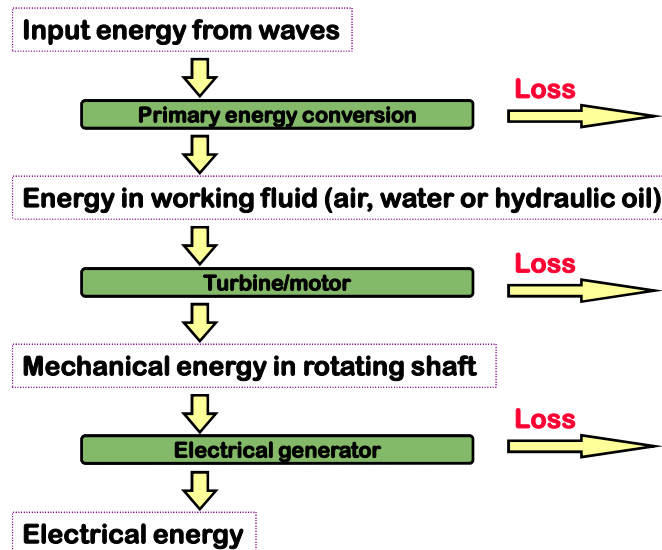
Nothing seems to happen with the wave!

If this device delivers energy, it seems to be a **perpetual engine machine!**

© 2008 SQUARET

Animation downloaded 2016-08-26 from <http://www.emec.org.uk/marine-energy/wave-devices>

## Schematic principle for extracting wave energy

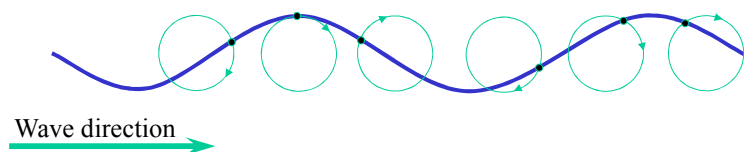


**These lectures are mainly aimed at  
understanding  
primary conversion of wave-energy**

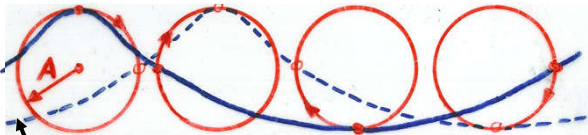
## What is a wave?

- Everyone has seen waves on lakes or oceans. Waves are actually a form of energy. Energy, not water, moves along the ocean's surface. The water particles only travel in small circles as a wave passes.

Snapshot of the water surface at a **certain instant**:



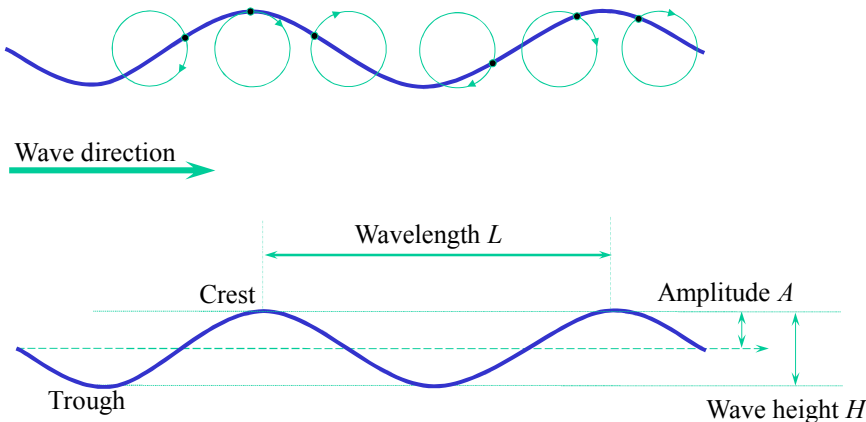
# GRAVITY WAVE ON DEEP WATER



One quarter period later

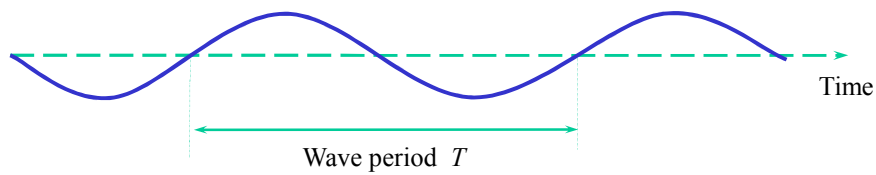
# How to describe a wave

Snapshot of the water surface at a certain instant:

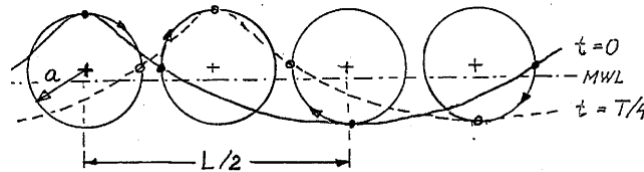


## Surface elevation versus time

At a fixed position in space:



$$\text{Frequency } f = 1 / T$$



A particular water particle moves once around its circle in a time  $T$  (the "period").

The troughs are wider than the crests.

The difference is negligible for small waves  
 $a \ll L/2\pi = 1/k$ .

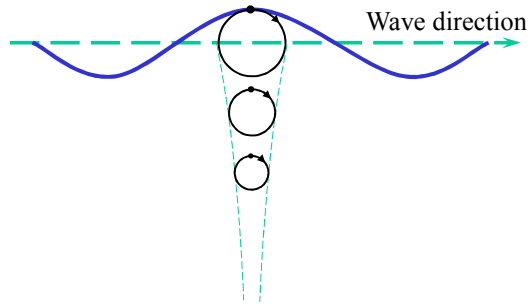
Then the water surface has a sinusoidal shape.

"Small waves" = "linear waves" (linear theory applicable)  
 $\lambda = L$  is the "wavelength"

$k = 2\pi/L$  is the "angular repetency" (or "wavenumber")

# What happens underwater?

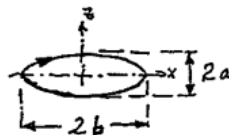
In deep water the water particles travel in vertical circles (while in shallow water the motion is elliptical)  
 This motion of water particles also happens underwater, but the particle velocity and thereby the circle radius decrease quickly (exponentially) as you go deeper in the water.



On deep water, the radius of the water-orbiting circle decays exponentially with the distance ( $-z$ ) below the mean water surface. According to the factor  $\exp\{-k(-z)\}$

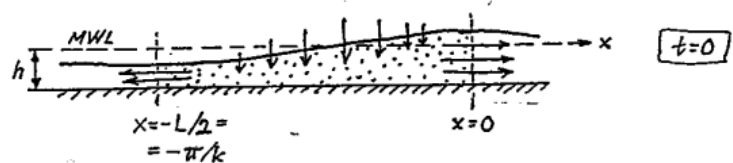
On shallow water:

- elliptical orbits

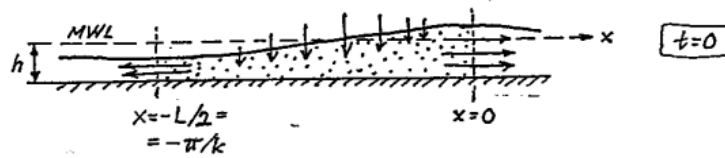


Excursion of water particle:  $\eta = s_z = a \cos(\omega t - kx)$   
 $s_x = b \sin(\omega t - kx)$

Velocity of water particle  $\dot{s}_z = v_z = -\omega a \sin(\omega t - kx)$   
 $\dot{s}_x = v_x = \omega b \cos(\omega t - kx)$







Water flow outwards:  $(v_x)_{\max} 2h = \omega b 2h$  (shallow water)

Water flow downwards:

$$-\int_{-L/2}^0 v_z|_{t=0} dx = \omega a \int_{-L/2}^0 \sin(-kx) dx = \omega a \left[ \frac{\cos(kx)}{k} \right]_{-L/2}^0$$

$$= \frac{\omega a}{k} (1 - (-1)) = \frac{2\omega a}{k} = \frac{\omega a L}{\pi}$$

Balance of flow:  $\omega b 2h = \omega a L / \pi$   $\frac{a}{b} = kh = \frac{2\pi h}{L}$

Here very shallow water has been assumed. (OK if  $h < L/20$ ).

Balance of flow:  $\omega b 2h = \omega a L / \pi$   $\frac{a}{b} = kh = \frac{2\pi h}{L}$

On very shallow water the maximum values of the water particle excursion and the water velocity have a ratio

$$a/b = \omega a / \omega b = kh = 2\pi h / L$$

between the vertical and horizontal components. ("shallow water" means  $kh \ll 1$ )

On deep water ( $kh \gg 1$ )

• circular orbits  $b = a$   
 $(v_x)_{\max} = (v_z)_{\max} = \omega a$

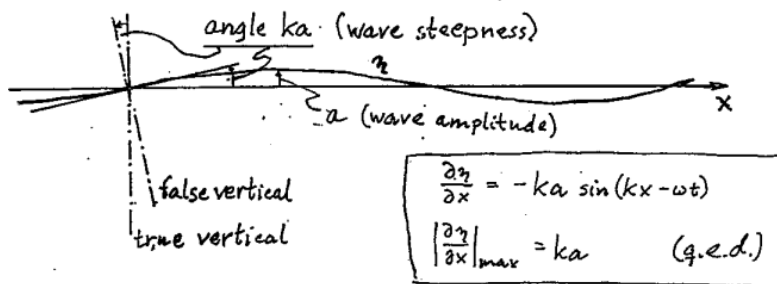
The phase velocity  $c = \omega/k$  for water waves:

Assume regular wave of small amplitude, that is of sinusoidal shape.

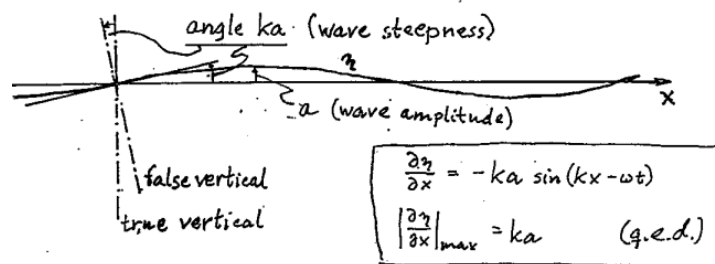
$$\eta = a \cos(kx - \omega t)$$

The maximum tilt (the "steepness") of the wave is

$$ka = 2\pi a/L$$



$$\eta = a \cos(kx - \omega t)$$



By "small" amplitude  $a$  we mean  $ka \ll 1$

$$\text{Then } \sin(ka) \approx \tan(ka) \approx ka$$

This is also angle between false vertical and true vertical

$$\frac{\text{horizontal acceleration}}{\text{gravity acceleration } (g)} = \text{tangent of angle of tilt}$$

$$\text{Maximum horizontal acceleration: } (i_x)_{\max} = g \tan(ka) \approx gka$$

$$\dot{s}_z = v_z = -\omega a \sin(\omega t - kx)$$

$$\dot{s}_x = v_x = \omega b \cos(\omega t - kx)$$

$\frac{\text{horizontal acceleration}}{\text{gravity acceleration (g)}} = \text{tangent of angle of tilt}$

Maximum horizontal acceleration:  $(\dot{v}_x)_{\max} = g \tan(ka) \approx gka$

From orbit motion:  $(\dot{v}_x)_{\max} = (\ddot{s}_x)_{\max} = \omega^2 b$



$$gka = \omega^2 b$$

$$\omega^2 = gk \frac{a}{b}$$

Phase velocity:  $c = \frac{L}{T} = \frac{\omega}{k} = \frac{g}{\omega} (a/b) = \sqrt{\frac{g}{k}} (a/b)$

The wavelength:  $L = 2\pi/k = \frac{g}{2\pi} T^2 (a/b)$

The period:  $T = 2\pi/\omega = \left\{ 2\pi \frac{L}{g} (b/a) \right\}^{1/2}$

For "deep" water (that is  $kh \gg 1$ ):

we have  $b/a = 1$  (circular orbit)

Then  $c = \frac{g}{\omega} = \left\{ \frac{g}{k} \right\}^{1/2} = \frac{g}{2\pi} T = \left\{ \frac{g}{2\pi} L \right\}^{1/2}$

$$L = cT = \frac{g}{2\pi} T^2$$

$$\frac{g}{2\pi} = \frac{9.81 \text{ m/s}^2}{2\pi} = 1.56 \text{ m/s}^2$$

Example:  $T = 10 \text{ s}$ :  $c = 15.6 \text{ m/s}$   $L = 156 \text{ m}$   
 $(= 56.2 \text{ km/h})$

The deep-water approximation is usually acceptable if  $h > L/3$  ( $kh > 2$ )

$$\frac{a}{b} = kh = \frac{2\pi h}{L}$$

For shallow water (that is  $kh \ll 1$ ):  
we found  $a/b = kh = 2\pi h/L$

$$\omega^2 = gk(a/b) = gk^2 h = ghk^2$$

$$c = \frac{L}{T} = \frac{\omega}{k} = \sqrt{gh}$$

$$L = cT = T\sqrt{gh}$$

Example:  $h = 6 \text{ m}$       $c = \sqrt{9.81 \cdot 6.0} = 7.7 \text{ m/s}$   
(= 27.6 km/h)

If  $T = 20 \text{ s}$       $L = 153 \text{ m}$

The shallow-water approximation is usually  
acceptable if  $h < L/20$  ( $kh < 0.3$ )

Superposition of two waves of the same L 11  
amplitude but slightly different frequencies:

$$\eta_1 = a \cos(\omega_1 t - k_1 x) \quad \eta_2 = a \cos(\omega_2 t - k_2 x)$$

$$\eta(x, t) = \eta_1(x, t) + \eta_2(x, t)$$

Recalling the trigonometric identity

$$\cos \alpha_1 + \cos \alpha_2 = 2 \cos \frac{\alpha_1 - \alpha_2}{2} \cos \frac{\alpha_1 + \alpha_2}{2}$$

$$\eta = 2a \cos\left(\frac{\omega_2 - \omega_1}{2} t - \frac{k_2 - k_1}{2} x\right) \cos\left(\frac{\omega_2 + \omega_1}{2} t - \frac{k_2 + k_1}{2} x\right)$$

$$\eta_1 = a \cos(\omega_1 t - k_1 x) \quad \eta_2 = a \cos(\omega_2 t - k_2 x)$$

$$\eta(x, t) = \eta_1(x, t) + \eta_2(x, t)$$

$$\eta = 2a \cos\left(\frac{\omega_2 - \omega_1}{2} t - \frac{k_2 - k_1}{2} x\right) \cos\left(\frac{\omega_2 + \omega_1}{2} t - \frac{k_2 + k_1}{2} x\right)$$

$$\text{Set } \omega_1 = \omega - \Delta\omega \quad \omega_2 = \omega + \Delta\omega$$

$$k_1 = k - \Delta k \quad k_2 = k + \Delta k$$

$$\eta = 2a \cos(\Delta\omega t - \Delta k x) \cos(\omega t - kx)$$

Varies slowly  
if  $\Delta\omega \ll \omega$  (and hence  $\Delta k \ll k$ )

Resulting wave of "angular frequency"  $\omega$   
and a slowly varying "amplitude"

$$2a \cos(\omega t - kx)$$

This "amplitude" propagates with a  
speed

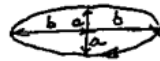
$$c_g = \frac{\Delta\omega}{\Delta k}$$

which (if  $\Delta\omega \rightarrow 0$ ) is termed the group velocity

$\frac{\text{horizontal acceleration}}{\text{gravity acceleration (g)}} = \text{tangent of angle of tilt}$

Maximum horizontal acceleration:  $(\dot{V}_x)_{\max} = g \tan(ka) \approx gka$

From orbit motion:  $(\dot{V}_x)_{\max} = (\ddot{x})_{\max} = \omega^2 b$



$$gka = \omega^2 b$$

$$\omega^2 = gk \frac{a}{b}$$

Phase velocity:  $c = \frac{L}{T} = \frac{\omega}{k} = \frac{g}{\omega} (a/b) = \sqrt{\frac{g}{k}} (a/b)$

The wavelength:  $L = 2\pi/k = \frac{g}{2\pi} T^2 (a/b)$

The period:  $T = 2\pi/\omega = \left\{ 2\pi \frac{L}{g} (b/a) \right\}^{1/2}$

We found  $\omega^2 = gk$  ( $a/b = 1$ ) (the "dispersion" equation)

On deep water ( $a/b = 1$ )  $\omega^2 = gk \Rightarrow 2\omega d\omega = g dk$

Phase velocity  $c = \frac{\omega}{k} = \frac{g}{\omega} = \sqrt{g/k}$

Group velocity  $c_g = \frac{d\omega}{dk} = \frac{g}{2\omega} = \frac{1}{2}c$

Thus we have the important result that on deep water the group velocity is half of the phase velocity

On very shallow water ( $a/b = kh$ )  $\omega = k\sqrt{gh}$

$c_g = \frac{d\omega}{dk} = \frac{\omega}{k} = \sqrt{gh} = c$

The group velocity and phase velocity are equal and independent of the frequency so long as the water may be considered to be shallow ( $\omega\sqrt{h/g} = kh \ll 1$ )

On **deep** water, the longer waves move faster than the shorter waves.

$$c = \frac{g}{\omega} = \left\{ \frac{g}{k} \right\}^{1/2} = \frac{g}{2\pi} T = \left\{ \frac{g}{2\pi} L \right\}^{1/2}$$

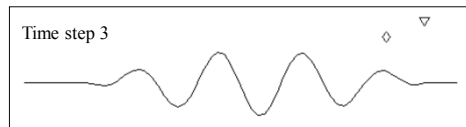
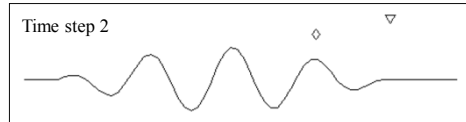
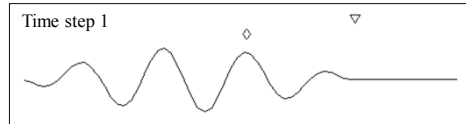
$$c_g = \frac{d\omega}{dk} = \frac{g}{2\omega} = \frac{1}{2}c$$



Photo: Magne Falnes, 1999

## Wave velocities

- The energy in the waves travel with the *group velocity*  $c_g$ . The individual waves travel faster - they are born on the rear end of the group, and they die in the front end. On deep water this *phase velocity* is twice the group velocity:



$$c = 2c_g = \frac{g}{2\pi} T = (1.56 \text{ m/s}^2) \cdot T$$

### Example:

Assume that the sea is calm. Then, suddenly a storm develops  $l=300 \text{ km}$  from land. How long time afterwards can we record swells of period  $T=14 \text{ s}$  at the shore? What if the period is  $T=10 \text{ s}$ ? Assume that the water depth is more than  $200 \text{ m}$ .

Solution:

Deep-water formulas are applicable because

$$L = 1,56 T^2 = 1,56 \cdot 14^2 = 306 \text{ m, that is } h > 200 \text{ m} > L/3 = 102 \text{ m}$$

The group velocity is

$$c_g = \frac{1}{2} c = \frac{1}{2} \frac{L}{T} = \frac{1}{2} 1,56 T = 0,78 T \\ = 0,78 \cdot 14 = 10,9 \text{ m/s}$$

Time before swell record

$$\Delta t = \frac{l}{c_g} = \frac{l}{0,78 T} = \frac{300 \cdot 10^3}{0,78 T} = \frac{3,8 \cdot 10^5}{T}$$

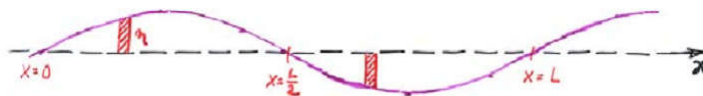
$$T = 14 \text{ s: } \Delta t = \frac{3,8 \cdot 10^5}{14} = 27 \cdot 10^3 \text{ s} = 7,6 \text{ hours}$$

$$T = 10 \text{ s} \quad \Delta t = \frac{3,8 \cdot 10^5}{10} = 38 \cdot 10^3 \text{ s} = 10,7 \text{ hours}$$

## Potential energy (averaged over time) for wave on sea surface.

Assumption: A propagating plane wave, sinusoidal in time and space.

Sea surface rectangle. Length: 1 wavelength  $L$ . Width: 1 length unit (1 m).



$$E_p L = \int_0^{L/2} \rho g \eta^2 \frac{1}{2} dx = \rho g \int_0^{L/2} \eta^2 dx = \rho g \frac{L}{2} \frac{1}{2} |\eta|_{\max}^2$$

Potential energy per unit area of the sea surface:  $E_p = \rho g \frac{1}{4} |\eta|_{\max}^2$

It can be shown [cf. Twidell + Weirt (2015), § 11.3.1] that there is an equal amount of kinetic energy associated with the moving water below this sea surface rectangle:

$$E_p L = E_k L$$

Total stored energy per unit area of the sea surface:

$$E = E_p + E_k = 2E_p = 2E_k = \frac{1}{2} \rho g |\eta|_{\max}^2$$



The wave-power level  $J$ : the flow of wave power per unit width of the wave front

Total stored energy per unit area of the sea surface:

$$E = E_p + E_k = 2E_p = 2E_k = \frac{1}{2} \rho g |\eta|_{\max}^2$$

$$[E] = J/m^2$$

Flow of energy per unit width of the wave front  $J$ :

$$J = c_g E = \frac{c_g}{2} \rho g |\eta|_{\max}^2 \quad c_g = \text{group velocity of the wave}$$

$$[J] = \frac{m}{s} \frac{J}{m^2} = \frac{W}{m}$$

For the case of deep water:  $c_g = c_f/2 = \frac{g}{2\omega} = \frac{gT}{4\pi}$

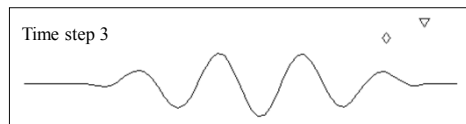
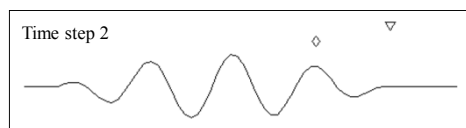
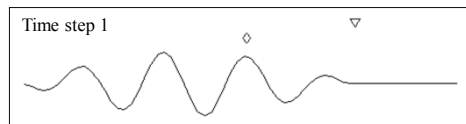
The «wave-power level»:

$$J = \frac{\rho g^2}{4\omega^2} |\eta|_{\max}^2 = \frac{\rho g^2 T}{8\pi} |\eta|_{\max}^2 = \frac{\rho g^2 T}{32\pi} H^2 = \left(976 \frac{W}{s \cdot m^2}\right) T H^2$$

The wave height:  $H = 2|\eta|_{\max}$

## Wave velocities

- The energy in the waves travel with the *group velocity*  $c_g$ . The individual waves travel faster - they are born on the rear end of the group, and they die in the front end. On deep water this *phase velocity* is twice the group velocity:



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The «wave-power level»:

$$J = \frac{\rho g^2}{4\omega} |\eta|_{\max}^2 = \frac{\rho g^2 T}{8\pi} |\eta|_{\max}^2 = \frac{\rho g^2 T}{32\pi} H^2 = \left(976 \frac{\text{W}}{\text{s}\cdot\text{m}^2}\right) T H^2$$

The wave height:  $H = 2|\eta|_{\max}$

Plane wave propagating on deep water in the positive x direction:  $\eta = A \cos(\omega t - kx + \alpha)$

$$J = \frac{\rho g^2}{4\omega} A^2 = \frac{\rho g^2}{8\pi f} A^2$$

$$E = 2E_p = 2E_k = \frac{1}{2} \rho g A^2 = \rho g \overline{\eta^2}$$

Multi-frequency sea wave:  $\eta = \sum_m A_m \cos(\omega_m t - k_m x + \alpha_m)$

$$J = \sum_m \frac{\rho g^2}{8\pi f_m} A_m^2 \quad E = \frac{1}{2} \rho g \sum_m A_m^2$$

Multi-frequency sea wave:  $\eta = \sum_m A_m \cos(\omega_m t - k_m x + \alpha_m)$

$$J = \sum_m \frac{\rho g^2}{8\pi f_m} A_m^2 \quad E = \frac{1}{2} \rho g \sum_m A_m^2$$

More general sea wave:  $\eta = \eta(x, y, t)$

$$E = \rho g \overline{\eta^2(x, y, t)} \equiv \rho g \int_0^\infty S(f) df$$

where we have introduced the real sea wave's «energy spectrum»  $S(f)$ , for which the SI unit is  $\text{m}^2/\text{Hz}$ . The overbar denotes time average.

Spectrally defined «significant wave height»:

$$H_{m0} = 4\sqrt{m_0} \quad m_0 \equiv \int_0^\infty S(f) df$$

$$\eta = \sum_m A_m \cos(\omega_m t - k_m x + \alpha_m)$$

$$J = \sum_m \frac{\rho g^2}{8\pi f_m} A_m^2 \quad E = \frac{1}{2} \rho g \sum_m A_m^2$$

$$\eta = \eta(x, y, t)$$

$$E = \rho g \overline{\eta^2(x, y, t)} \equiv \rho g \int_0^\infty S(f) df$$

Spectrally defined «significant wave height»:

$$H_{m0} = 4 \sqrt{m_0} \quad m_0 \equiv \int_0^\infty S(f) df$$

Spectrally defined «wave-power level»:

$$J = \frac{\rho g^2}{4\pi} \int_0^\infty \frac{S(f)}{f} df = \frac{\rho g^2}{4\pi} m_{-1}$$

$$m_{-1} \equiv \int_0^\infty f^{-1} S(f) df$$

Spectrally defined «significant wave height»:

$$H_{m0} = 4 \sqrt{m_0} \quad m_0 \equiv \int_0^\infty S(f) df$$

Spectrally defined «wave-power level»:

$$J = \frac{\rho g^2}{4\pi} \int_0^\infty \frac{S(f)}{f} df = \frac{\rho g^2}{4\pi} m_{-1}$$

$$m_{-1} \equiv \int_0^\infty f^{-1} S(f) df$$

Spectral moment of order  $j$ :  $m_j \equiv \int_0^\infty f^j S(f) df$

Spectrally defined  
«energy period»:

$$T_j = T_{-1,0} \equiv \frac{m_{-1}}{m_0}$$

and «wave-  
power level»:

$$J = \frac{\rho g^2}{4\pi} T_j m_0 = \frac{\rho g^2}{64\pi} T_j H_{m0}^2$$

## Wind waves and swells

- Waves generated by wind are called *wind waves*. When the waves propagate outside their region of generation, they are called *swells* [in Norwegian: *dønning*]. Where the water is deep, swells can travel very large distances, for instance across oceans, almost without loss of energy.

On **deep** water, the longer waves move faster than the shorter waves.

$$c = \frac{g}{\omega} = \left\{ \frac{g}{k} \right\}^{1/2} = \frac{g}{2\pi} T = \left\{ \frac{g}{2\pi} L \right\}^{1/2} \quad c_g = \frac{d\omega}{dk} = \frac{g}{2\omega} = \frac{1}{2} c$$

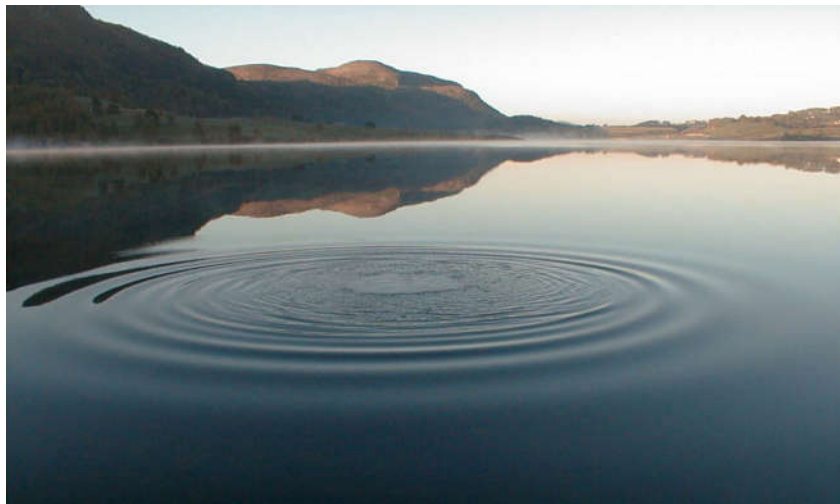
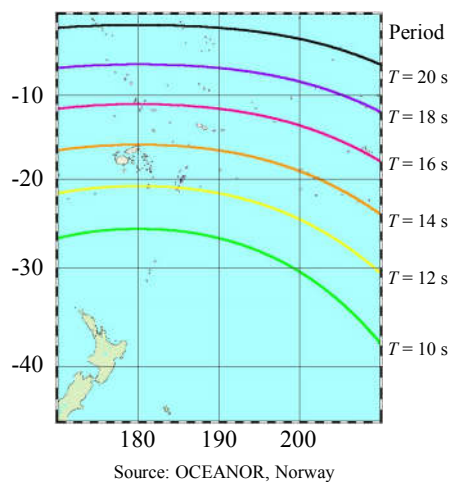


Photo: Magne Falnes, 1999

## Swells propagating across the Pacific



- Since the group velocity is proportional to the period, low-frequency waves move faster away from a storm centre than high-frequency waves. The figure shows the situation 4 days after a storm with centre located at 170° east and 50° south.

The wave-power level  $J$ : the flow of wave power per unit width of the wave front

Total stored energy per unit area of the sea surface:

$$E = E_p + E_k = 2E_p = 2E_k = \frac{1}{2} \rho g |\eta|_{\max}^2$$

$$[E] = J/m^2$$

Flow of energy per unit width of the wave front  $J$ :

$$J = c_g E = \frac{\rho g}{2} |\eta|_{\max}^2 c_g \quad c_g = \text{group velocity of the wave}$$

$$[J] = \frac{m}{s} \frac{J}{m^2} = \frac{W}{m}$$

For the case of deep water:  $c_g = c_f/2 = \frac{g}{2\omega} = \frac{gT}{4\pi}$

The «wave-power level»:

$$J = \frac{\rho g^2}{4\omega} |\eta|_{\max}^2 = \frac{\rho g^2 T}{8\pi} |\eta|_{\max}^2 = \frac{\rho g^2 T}{32\pi} H^2 = \left(976 \frac{W}{s \cdot m^2}\right) T H^2$$

The wave height:  $H = 2|\eta|_{\max}$

## Energy content of waves

- For a sinusoidal wave of height  $H$ , the average energy  $E$  stored on a horizontal square metre of the water surface is:

$$E = k_E H^2$$

$$k_E = \rho g / 8 = 1.25 \text{ kW} \cdot \text{s/m}^4$$

$$\rho = \text{mass density of sea water} \approx 1020 \text{ kg/m}^3$$

$$g = \text{acceleration of gravity} \approx 9.8 \text{ m/s}^2$$

- Half of this is potential energy due to water lifted from wave troughs to wave crests. The remaining half is kinetic energy due to the motion of the water.

$$\text{Example: } H = 2\text{m} \Rightarrow E = 5 \text{ kW} \cdot \text{s/m}^2$$

## The wave-power level

- The “wave-power level” (energy transport per metre width of the wave front) is

$$J = c_g E$$

On deep water the group velocity is  $c_g = gT/4\pi$ , which gives

$$J = k_J T H^2$$

$$k_J = \rho g^2 / 32 \pi \approx 1 \text{ kW/m}^3\text{s}$$

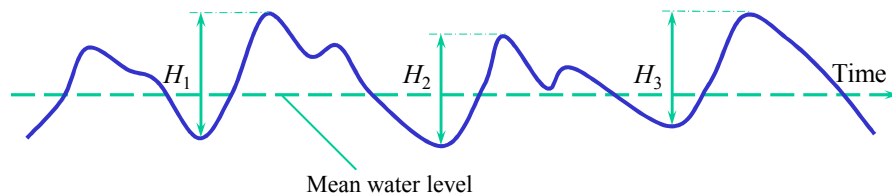
Example:

$$T = 10\text{s and } H = 2\text{m} \Rightarrow J = 40 \text{ kW/m}$$

## Significant wave height

The real-sea wave height parameter is the *significant wave height*. It is traditionally defined as the average of the highest one third of the individual trough-to-crest heights  $H_i$  ( $i=1,2,3,\dots$ ), and is denoted by  $H_{1/3}$ .

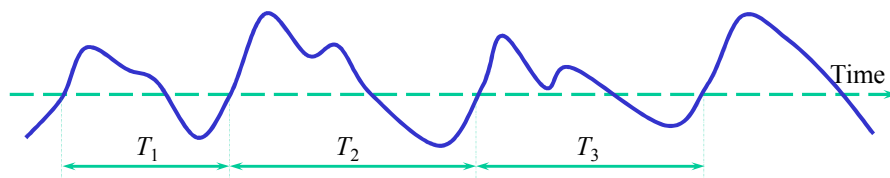
$$H_{1/3} = \frac{H_1 + H_2 + \dots + H_{N/3}}{N/3}$$



## Average zero up-cross time $T_z$

- The individual *zero up-cross time*  $T_i$  is the time interval between two consecutive instants where the wave elevation crosses the zero level in the upward direction. An average of these over a certain time provides a useful measure of the real-sea wave period.

$$T_z = \frac{T_1 + T_2 + \dots + T_N}{N}$$



## Wave spectrum

- A quantity derived from wave measurements is the so-called energy spectrum  $S(f)$ . It tells us how much energy is carried by the different frequency components in the real-sea “mixture” of waves. For a sinusoidal wave the average stored energy was given by

$$E = \rho g H^2 / 8$$

- For a real sea wave we have instead

$$E = \rho g \int_0^{\infty} S(f) df \equiv \rho g H_s^2 / 16$$

$$\eta = \sum_m A_m \cos(\omega_m t - k_m x + \alpha_m)$$

$$J = \sum_m \frac{\rho g^2}{8\pi f_m} A_m^2 \quad E = \frac{1}{2} \rho g \sum_m A_m^2$$

$$\eta = \eta(x, y, t)$$

$$E = \rho g \overline{\eta^2(x, y, t)} \equiv \rho g \int_0^{\infty} S(f) df$$

Spectrally defined «significant wave height»:

$$H_{m0} = 4 \sqrt{m_0} \quad m_0 \equiv \int_0^{\infty} S(f) df$$

Spectrally defined «wave-power level»:

$$J = \frac{\rho g^2}{4\pi} \int_0^{\infty} \frac{S(f)}{f} df = \frac{\rho g^2}{4\pi} m_{-1}$$

$$m_{-1} \equiv \int_0^{\infty} f^{-1} S(f) df$$



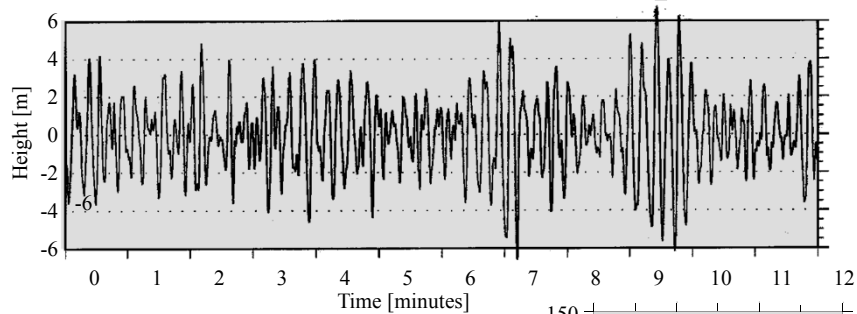
## Wave-power level in terms of significant wave height

$$\int_0^{\infty} S(f) df \equiv H_s^2 / 16$$

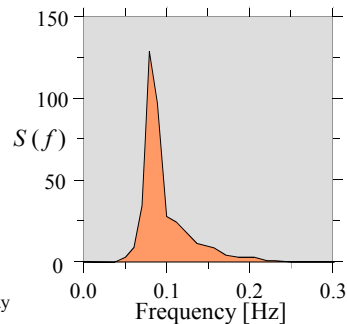
• Here  $H_s$  is the modern definition of significant wave height, which in practice agrees quite well with our previous definition  $H_{1/3}$ . Another quantity, the so-called wave energy period  $T_J$ , may be derived from the wave spectrum  $S(f)$ . The wave-power level by real sea waves is now calculated by

$$J = (k_J / 2) T_J H_s^2 \quad k_J / 2 \approx 0.5 \text{ kW/s m}^3$$

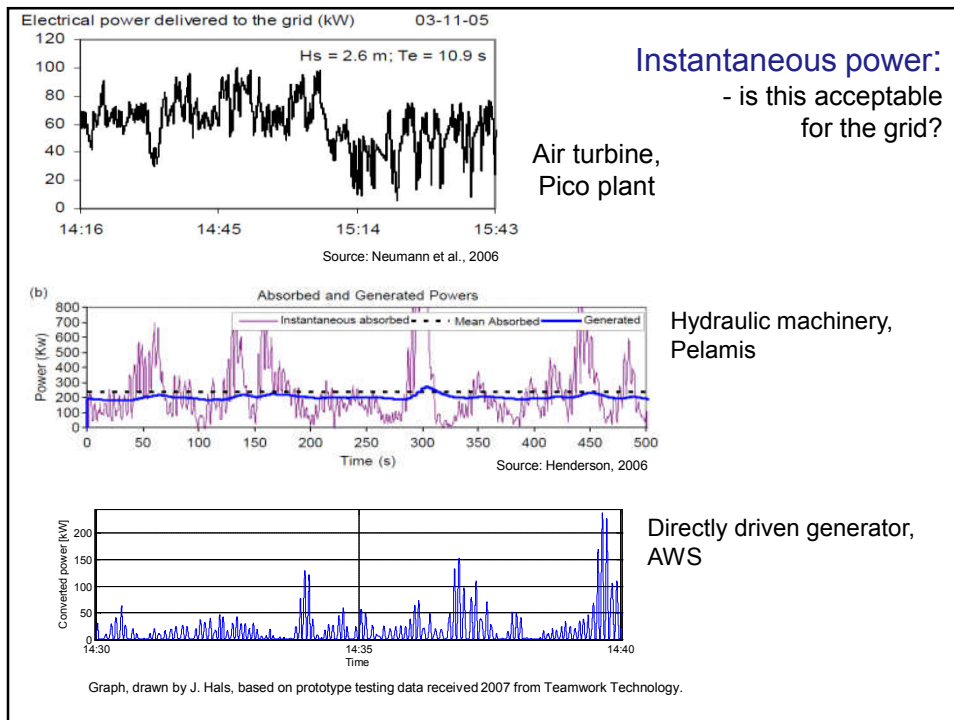
## A measurement example



• This time series (above) from high sea shows that individual waves vary greatly in size and form. The corresponding energy spectrum is shown to the right. For this storm wave the significant wave height is  $H_s = 8$  m.

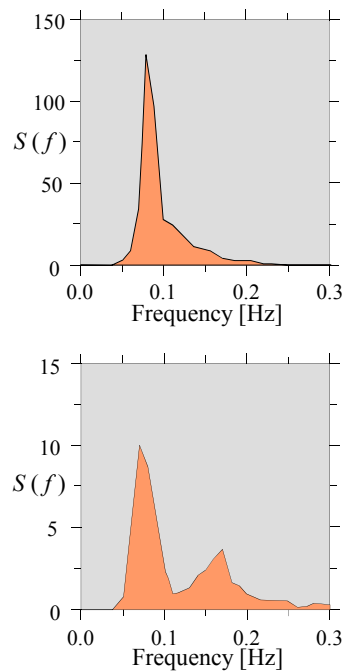


Source: OCEANOR, Norway

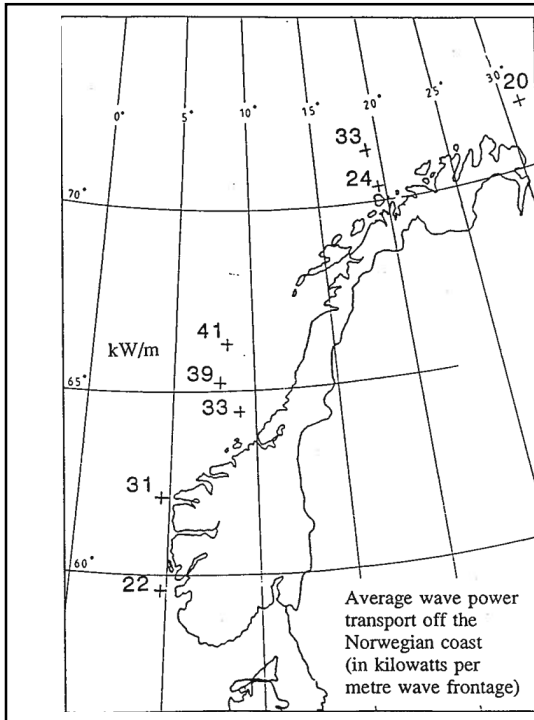


## Real-sea spectra

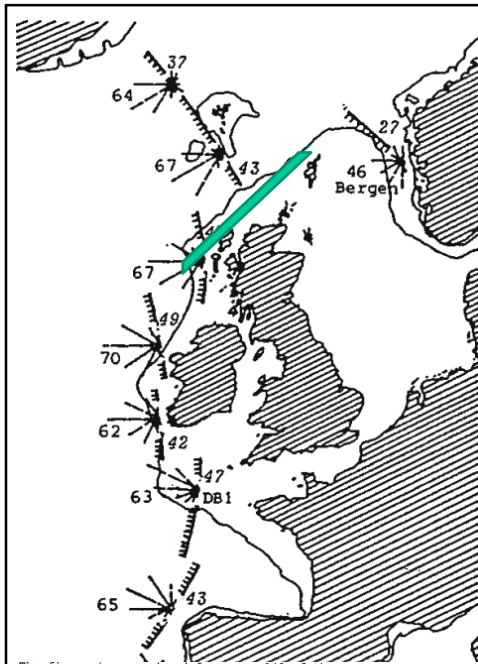
- These are typical energy spectra from wind-sea conditions (top) and mixed wind-sea and swell conditions (bottom).
- The swell contains lower frequencies (high peak) than the the wind waves (low peak).
- Significant wave heights: 8 m (top) and 3 m (bottom)



Source: OCEANOR, Norway



Norwegian wave-power-level [in kW per m wavefront]. (Torsethaugen 1990).



The figure is reproduced from page 148 of the paper  
 Hollison, D.: "Wave climate and the wave power resource". *Proceedings IUTAM Symposium on Hydrodynamics of Ocean Wave-Energy Utilization*, (edited by D.V. Evans and A.F. de Falcão) Lisbon, Portugal, 8-11 July 1985. Springer-Verlag, pp. 132-156, 1986.

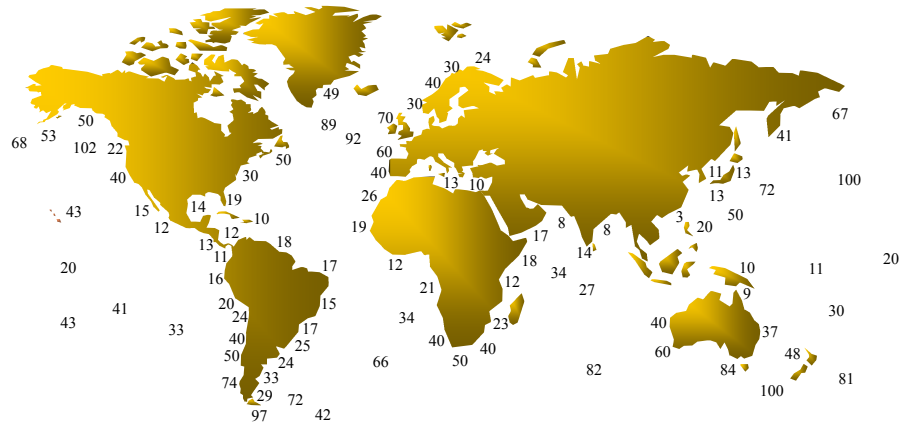
Who (which nations) have the propriety right to exploit the oceans' wave energy?

If wave energy is being exploited by WEC arrays ranging from north of Shetland to south of the Hebrides, there may be reduced wave energy to exploit at the west coast of Denmark and Norway.

**Make international agreements before wave-energy has a commercial interest!**

Could any country, e.g. Switzerland, exploit wave energy by floating WEC arrays in international waters of the Atlantic?

## Distribution of wave energy transport



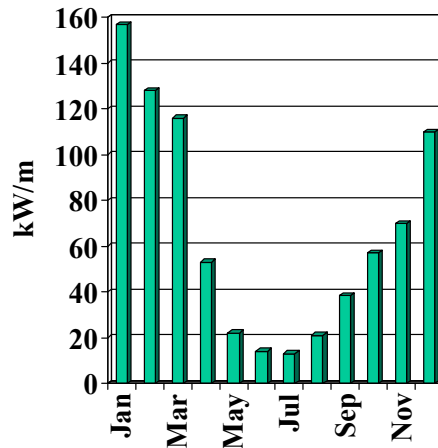
Average wave power levels are approximate and given in kW/m of the wave front.

## Seasonal variation

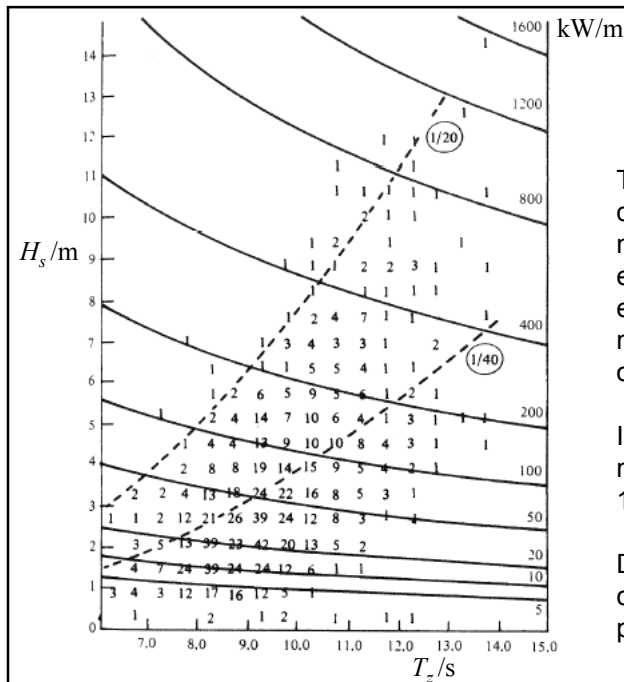
- The average values of wave-energy transport vary somewhat from one year to next year. The values vary more between seasons. On the northern hemisphere, the average values for November and May may differ by a factor of two or more. There is significantly more wind energy and wave energy in winter than in summer, although it is opposite for solar energy. Because there may be waves (swells) even in the absence of wind, wave energy is more persistent than wind energy.

## Seasonal variation at (57° N, 9° W )

- The chart shows the seasonal variation of wave energy transport at a measurement site close to Barra in the Hebrides off the Scottish coast. The annual average for the shown year was 65 kW/m.



Based on WERATLAS, European Wave Energy Atlas, 1996



### “Scatter”-diagram

The numbers on the graph denote the average numbers of occurrences of each  $H_s$ - $T_z$  combination for each 1000 wave measurements made over one year.

Increasing curves indicate maximum wave steepness 1/40 and 1/20.

Declining curves indicate constant values of wave-power level in kW/m.

Figure after Ian Glendenning 1978 (cf. book # D6 in the list: [http://folk.ntnu.no/istnes/wave\\_books/wave\\_energy.htm](http://folk.ntnu.no/istnes/wave_books/wave_energy.htm)).

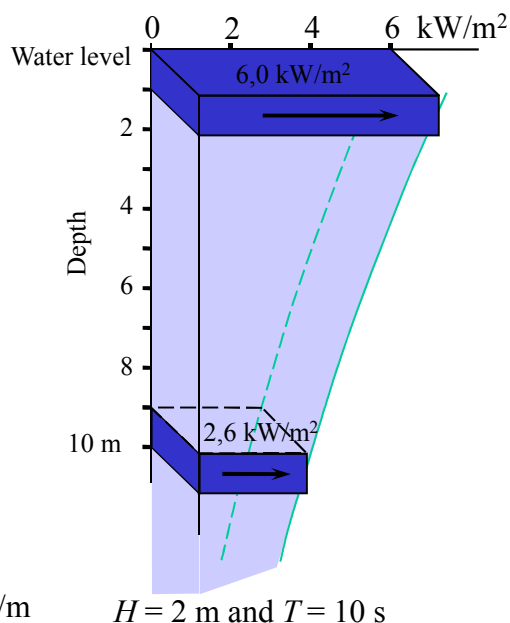
Source : Ian Glendenning, 1977

- Average energy intensity:
- Solar energy: 100 - 200 W/m<sup>2</sup>
- Wind energy: 400 - 600 W/m<sup>2</sup>
- Wave energy: 4 - 6 kW/m<sup>2</sup>  
(just below the sea surface)

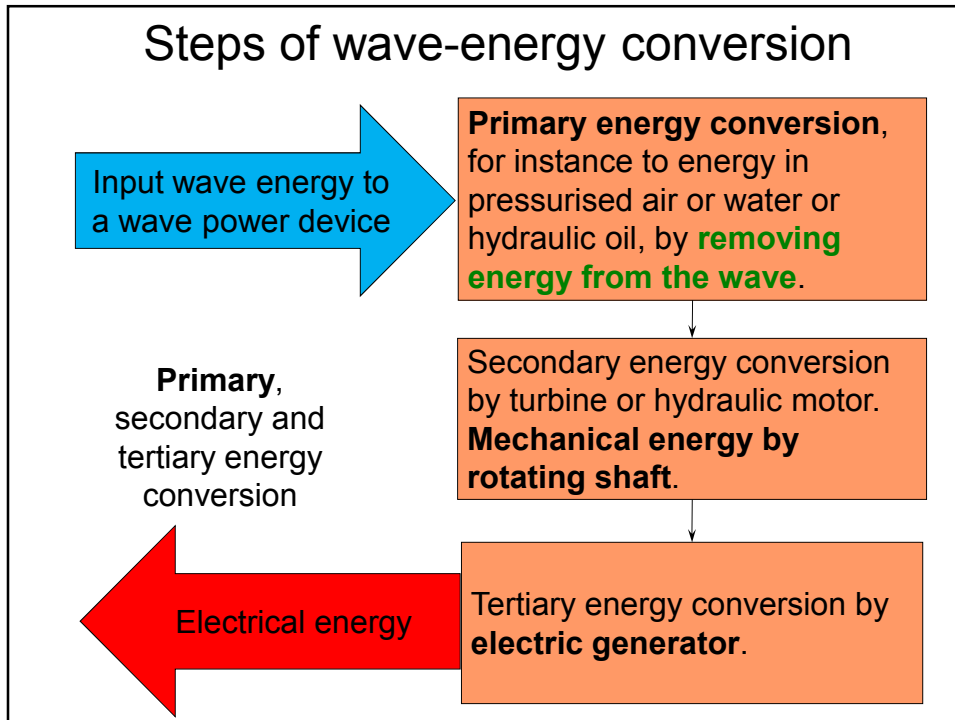
### Vertical distribution of wave-energy transport

- As we have seen, the water particles move in circles with decreasing radius in the depth. Consequently, the energy flow density decreases as we go deeper in the water. In fact, on deep water, 95 % of the energy transport takes place between the surface and the depth  $L/4$ . ( $L$  is the wavelength).

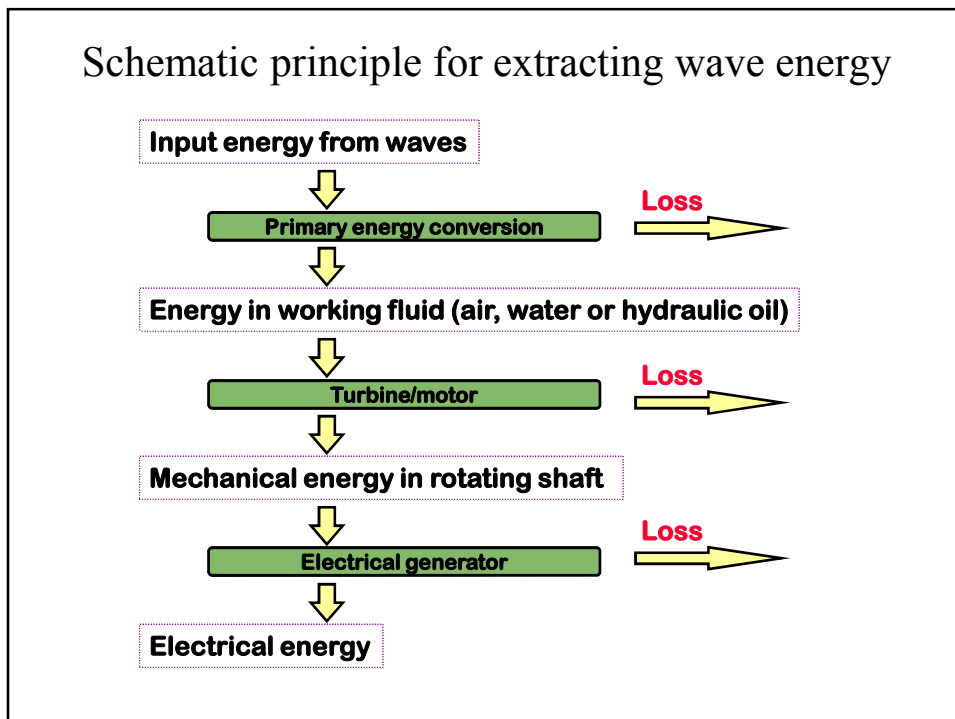
$$J = 40 \text{ kW/m}$$



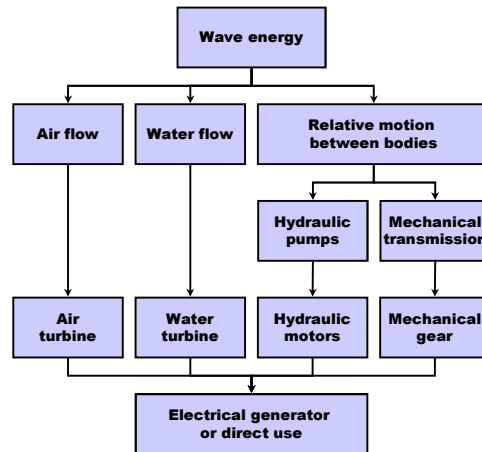
## Steps of wave-energy conversion



## Schematic principle for extracting wave energy



## Power take-off alternatives



The length size  $D$  of a wave-energy converter (WEC) compared to one wavelength  $L$ .

**Terminator:**  $D$  approx. equal to or larger than  $L$ .

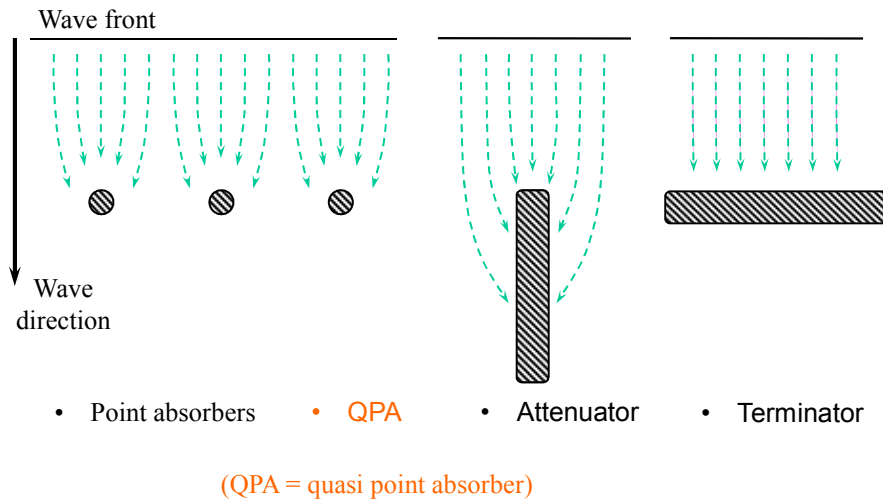
**Attenuator:**  $D$  approx. equal to or larger than  $L$ .

**Point absorber (PA):**  $D$  approx. equal to or smaller than  $L/10$ .

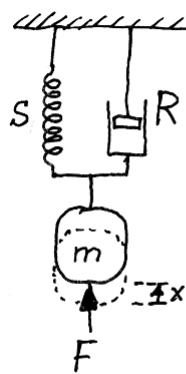
**«Quasi point absorber» (QPA):** Size between point absorber and «line absorber» (terminator or attenuator).



## Classification of WECs - According to size and orientation



### Mechanical oscillator



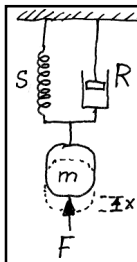
Mass  $m$  displaced a distance  $x$  from equilibrium position

Forces:

$F$  = external applied force

$F_s = -Sx$  spring force

$F_R = -R\dot{x}$  damping force



For mathematical simplicity the damping force is assumed proportional to the velocity  $\dot{x} = \frac{dx}{dt}$

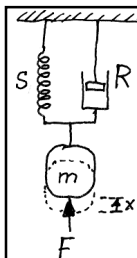
$R$  = "mechanical resistance"

The spring force is assumed proportional to the displacement  $x$  from equilibrium.

$$\text{Newton: } m\ddot{x} = F + F_S + F_R = F - Sx - R\dot{x}$$

$$m\ddot{x} + R\dot{x} + Sx = F \quad (2)$$

(External force balanced against 1) inertial force  
2) damping force and 3) spring force)



$$m\ddot{x} + R\dot{x} + Sx = F \quad (2)$$

(External force balanced against 1) inertial force  
2) damping force and 3) spring force)

Energy = Force  $\times$  Displacement

Power = Force  $\times$  Velocity

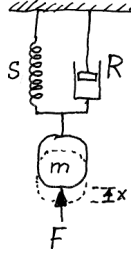
Power applied to the mechanical system

$$P = F\dot{x} = m\ddot{x}\dot{x} + R\dot{x}^2 + Sx\dot{x} =$$

$$= m\dot{x} \frac{d\dot{x}}{dt} + R\dot{x}^2 + S \frac{dx}{dt} x =$$

$$= \frac{d}{dt} \left( \frac{1}{2} m \dot{x}^2 \right) + R\dot{x}^2 + \frac{d}{dt} \left( \frac{1}{2} S x^2 \right)$$

$$= R\dot{x}^2 + \frac{d}{dt} (W_k + W_p)$$



Power applied to the mechanical system

$$\begin{aligned}
 P &= F \dot{x} = m \ddot{x} \dot{x} + R \dot{x}^2 + S x \dot{x} = \\
 &= m \dot{x} \frac{d\dot{x}}{dt} + R \dot{x}^2 + S \frac{dx}{dt} x = \\
 &= \frac{d}{dt} \left( \frac{1}{2} m \dot{x}^2 \right) + R \dot{x}^2 + \frac{d}{dt} \left( \frac{1}{2} S x^2 \right) \\
 &= R \dot{x}^2 + \frac{d}{dt} (W_k + W_p)
 \end{aligned}$$

$W_k = \frac{1}{2} m \dot{x}^2 = \text{kinetic energy}$   
 $W_p = \frac{1}{2} S x^2 = \text{potential energy (of the spring)}$

POWER AND ENERGY RELATIONS. (6)

Mechanical power (rate of work) delivered by supplying the external force  $F(t)$ :

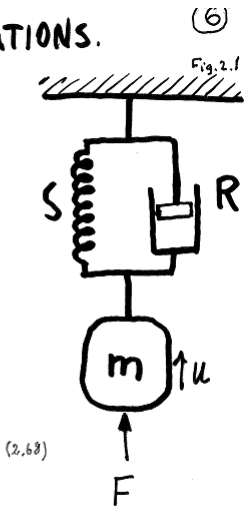
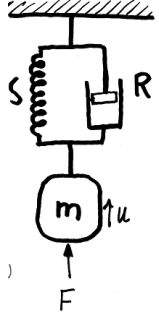


Fig. 2.1

$$\begin{aligned}
 P(t) &= F(t) u(t) = F(t) \dot{x}(t) \\
 F(t) &= F_m(t) - F_R(t) - F_S(t) \\
 &= m a(t) + R u(t) + S x(t)
 \end{aligned}$$

(2.68)

$$\underline{P(t) = P_R(t) + (P_k(t) + P_p(t))}$$



$$P(t) = P_R(t) + (P_k(t) + P_p(t))$$

$$P_R(t) = -F_R(t) u(t) = R \dot{u}^2 \quad (2.70)$$

$$P_k(t) = F_m(t) u(t) = m \ddot{u} u = \frac{d}{dt} W_k(t)$$

$$W_k(t) = \frac{1}{2} m (\dot{u}(t))^2 \quad \text{-kinetic energy.}$$

$$P_p(t) = -F_s(t) u(t) = S x \dot{x} = \frac{d}{dt} W_p(t) \quad (2.72)$$

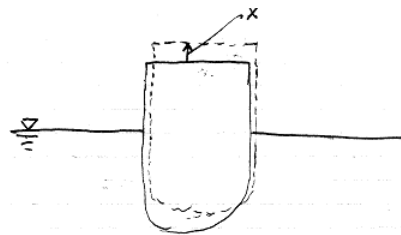
$$W_p(t) = \frac{1}{2} S (x(t))^2 \quad \text{-potential energy.}$$

Energy stored in the oscillating system:  

$$W(t) = W_k(t) + W_p(t) \quad (2.75)$$

$$P_k(t) + P_p(t) = \frac{d}{dt} W(t) \quad (2.76)$$

### Hydrostatic stiffness $S$ of buoyant body



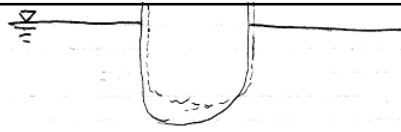
Water plane area  $A_w$ .  
 If body is lifted up a distance  $x$  from its equilibrium position, the upward buoyancy force is reduced by  $\rho g A_w x$ . Thus there

will be a restoring force

$$F_s = -\rho g A_w x$$

Restoring in a direction to restore the equilibrium

(Note: position and force are chosen to be positive upwards)



distance  $x$  from its equilibrium position, the upward buoyancy force is reduced by  $\rho g A_w x$ . Thus there

will be a restoring force

$$F_s = -\rho g A_w x$$

acting in a direction to restore the equilibrium

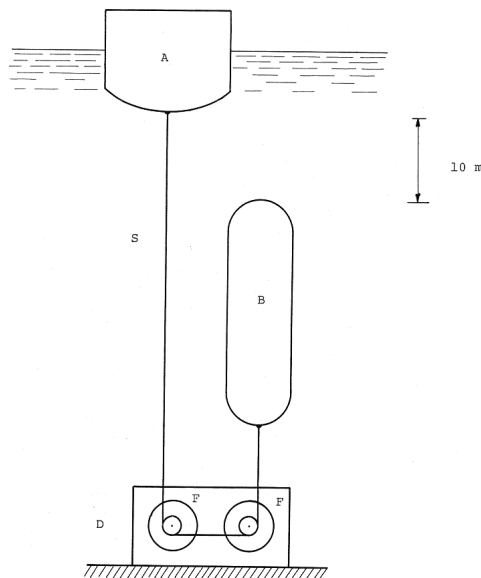
(Note: position and force are chosen to be positive upwards)

$$F_s = -Sx$$

Hydrostatic stiffness (buoyancy stiffness)  $S = \rho g A_w$

If the floating body is axisymmetric and of radius  $a$  (diameter  $2a$ ) the "water plane area" is  $A_w = \pi a^2$

$$S = \rho g \pi a^2$$



The hydrostatic buoyancy stiffness of floating body A provides **storage of potential energy**.

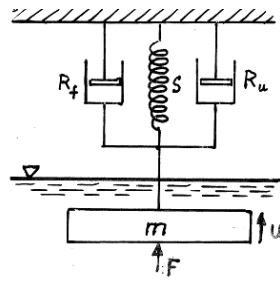
The mass of bodies A and B, as well as the two flywheels F, provides **storage of kinetic energy**.

Pumps or generators connected to the rotating flywheels may serve as **receivers of useful energy**.

Wave-power converter

[proposed by K. Budal 1974]

### Mechanical oscillator interacting with waves.



Mass  $m$  arranged to oscillate in water

It has an equilibrium position determined by a spring (e.g. hydrostatic "spring" for a floating body arranged to oscillate)

Mechanical resistances:  $R_f$   $R_u$   $R_r$

$\bar{P}_f = R_f \bar{u}^2$  = power lost by friction (including viscous losses in the water, etc.)

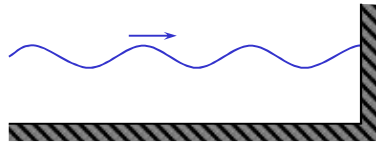
$\bar{P}_u = R_u \bar{u}^2$  = useful converted power

$\bar{P}_r = R_r \bar{u}^2$  = radiated power (if a body in water oscillates, a wave is generated which carries away wave energy.)

## A paradox?

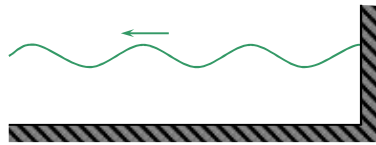
- Absorption of wave energy from the sea may be considered as a phenomenon of wave interference. Then wave energy absorption may be described by an apparently paradoxical statement:
  - *To absorb a wave means to generate a wave*
- or, in other words:
  - *To destroy a wave is to create a wave.*

Incident wave + reflected wave = standing wave



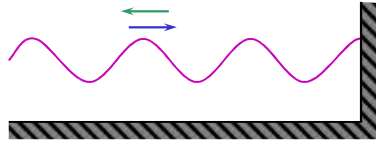
- Incident wave

+



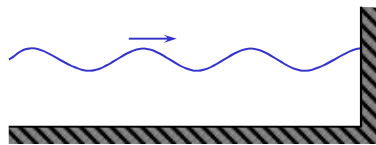
- Wave reflected from fixed wall

=



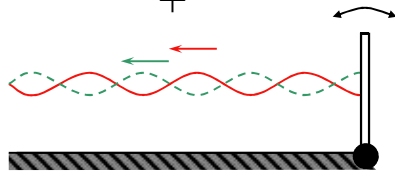
- Interference result: Standing wave composed of incident wave and reflected wave

“To absorb a wave means to generate a wave”  
- or “to destroy a wave means to create a wave”.



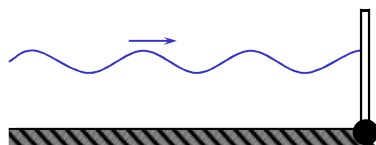
- Incident wave

+



- Wave reflected from fixed wall
- Wave generation on otherwise calm water (due to wall oscillation)

=



- The incident wave is absorbed by moving wall because the reflected wave is cancelled by the generated wave.

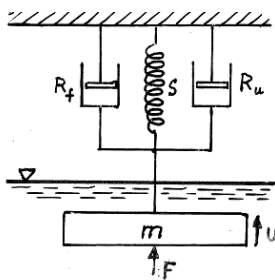
In this simple example, at optimum radiated-wave generation, the maximum absorbed energy equals 100 percent of the incident wave energy. Note also that the required, optimum, radiated wave has the same amplitude as the incident wave. Thus,

$$P_{r,OPT} = P_{a,MAX}$$

Observe that, in order to absorb, from the sea, the theoretically maximum wave power, **it is necessary that the wave-absorbing oscillating system, at optimum, has an ability to radiate as much power as the theoretically maximum absorbed power.**

This statement is valid also for systems of different geometrical configurations, where the maximum absorbed power is less than 100 percent of the incident wave power, provided **the required optimum oscillation can be realised**, that is, when no physical amplitude limitation, or other constraint, prevents the desired radiated wave from being realised.

### Mechanical oscillator interacting with waves.



Mass  $m$  arranged to oscillate in water

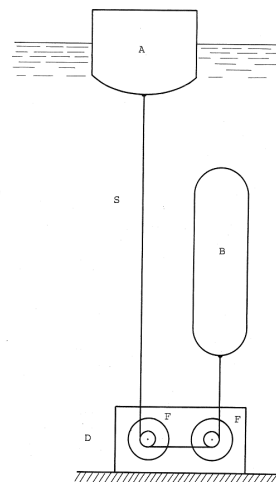
It has an equilibrium position determined by a spring (e.g. hydrostatic "spring" for a floating body arranged to oscillate).

Mechanical resistances:  $R_f$   $R_u$   $R_r$

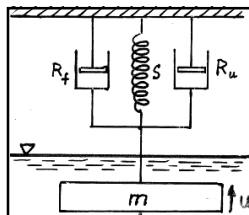
$\bar{P}_f = R_f \bar{u}^2$  = power lost by friction (including viscous losses in the water, etc.)

$\bar{P}_u = R_u \bar{u}^2$  = useful converted power

$\bar{P}_r = R_r \bar{u}^2$  = radiated power (if a body in water oscillates, a wave is generated ~~which carries away wave energy.~~)



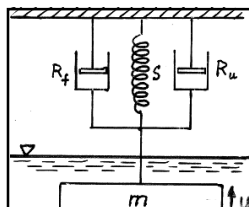




$\bar{P}_r = R_r \bar{u}^2 = \text{radiated power (if a body in water oscillates, a wave is generated which carries away wave energy.)}$

$R_f = \text{loss resistance}$   
 $R_u = \text{(useful) load resistance (e.g. a pump providing fluid to a turbine)}$   
 $R_r = \text{radiation resistance}$

Not only the mass  $m$  but also the surrounding water attain velocity (and acceleration). Hence an added hydrodynamic mass  $m_r$  is included in the dynamic system.

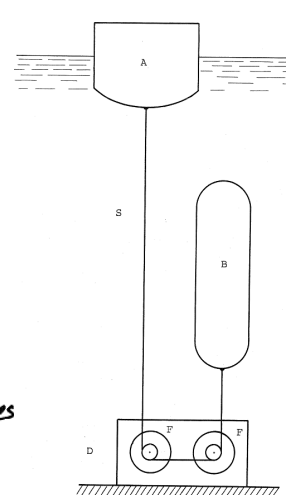
$$m \ddot{x} + m_r \ddot{x} + R_r \dot{x} + R_u \dot{x} + R_f \dot{x} + Sx = F_e$$


$m \ddot{x} + m_r \ddot{x} + R_r \dot{x} + R_u \dot{x} + R_f \dot{x} + Sx = F_e$

$F_e$  is the wave excitation force (that is, the wave force which the body experiences if it is not moving)

$[m \ddot{x} + R_u \dot{x} + R_f \dot{x} + Sx = F_e - m_r \ddot{x} - R_r \dot{x} \equiv F_w$

$F_w$  is the total wave force when the body oscillates. It is due to the total wave including the radiated wave which is generated by the body's motion.



$$m\ddot{x} + m_r\ddot{x} + R_r\dot{x} + R_u\dot{x} + R_f\dot{x} + Sx = F_e$$

$F_e$  is the wave excitation force (that is, the wave force which the body experiences if it is not moving)

Useful power

$$\overline{P_u} = R_u \overline{u^2} = R_u \frac{1}{2} u_0^2 \quad u = \dot{x}$$

$$R_u \dot{x} = F_e - R_n \dot{x} - R_f \dot{x} - (m + m_r) \ddot{x} - Sx$$

$$\begin{aligned} R_u \dot{x}^2 &= F_e \dot{x} - R_n \dot{x}^2 - R_f \dot{x}^2 - \underbrace{(m + m_r) \dot{x} \ddot{x} - Sx \dot{x}}_{-\frac{m + m_r}{2} \frac{d}{dt} \dot{x}^2 - \frac{S}{2} \frac{d}{dt} x^2} \\ &= -\frac{d}{dt} (W_{kinetic} + W_{potential}) \end{aligned}$$

Useful power

$$\overline{P_u} = R_u \overline{u^2} = R_u \frac{1}{2} u_0^2 \quad u = \dot{x}$$

$$R_u \dot{x} = F_e - R_n \dot{x} - R_f \dot{x} - (m + m_r) \ddot{x} - Sx$$

$$\begin{aligned} R_u \dot{x}^2 &= F_e \dot{x} - R_n \dot{x}^2 - R_f \dot{x}^2 - \underbrace{(m + m_r) \dot{x} \ddot{x} - Sx \dot{x}}_{-\frac{m + m_r}{2} \frac{d}{dt} \dot{x}^2 - \frac{S}{2} \frac{d}{dt} x^2} \\ &= -\frac{d}{dt} (W_{kinetic} + W_{potential}) \end{aligned}$$

Time-average value

$$\overline{P_u} = R_u \overline{\dot{x}^2} = \overline{F_e \dot{x}} - R_n \overline{\dot{x}^2} - R_f \overline{\dot{x}^2}$$

$$F_e \dot{x} = F_e u = F_0 \cos(\omega t) u_0 \cos(\omega t - \varphi)$$

$$\cos \alpha_1 \cos \alpha_2 = \frac{1}{2} \cos(\alpha_2 - \alpha_1) + \frac{1}{2} \cos(\alpha_1 + \alpha_2)$$

$$F_e \dot{x} = F_0 u_0 \left( \frac{1}{2} \cos \varphi + \frac{1}{2} \cos(2\omega t - \varphi) \right)$$

Time-average value

$$\overline{P}_m = R_m \overline{\dot{x}^2} = \overline{F_e \dot{x}} - R_n \overline{\dot{x}^2} - R_f \overline{\dot{x}^2}$$

$$F_e \dot{x} = F_e u = F_0 \cos(\omega t) u_0 \cos(\omega t - \varphi)$$

$$\cos \alpha_1 \cos \alpha_2 = \frac{1}{2} \cos(\alpha_2 - \alpha_1) + \frac{1}{2} \cos(\alpha_1 + \alpha_2)$$

$$F_e \dot{x} = F_0 u_0 \left( \frac{1}{2} \cos \varphi + \frac{1}{2} \cos(2\omega t - \varphi) \right)$$

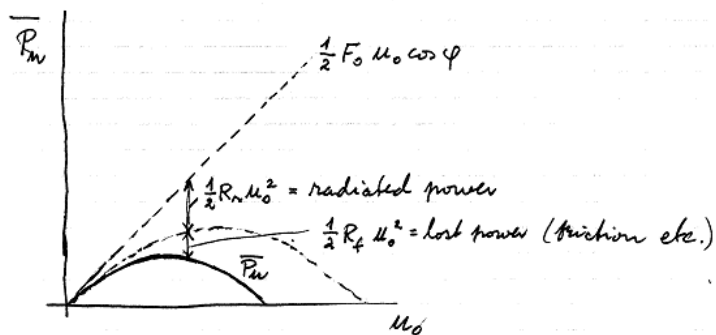
$$\overline{F_e \dot{x}} = \frac{1}{2} F_0 u_0 \cos \varphi$$

$$R_n \overline{\dot{x}^2} = \frac{1}{2} R_n u_0^2$$

$$\overline{P}_m = \frac{1}{2} F_0 u_0 \cos \varphi - \frac{1}{2} R_n u_0^2 - \frac{1}{2} R_f u_0^2$$

$$\overline{P}_m = \frac{1}{2} F_0 u_0 \cos \varphi - \frac{1}{2} R_n u_0^2 - \frac{1}{2} R_f u_0^2$$

How does  $\overline{P}_m$  depend on oscillation amplitude  $u_0$ ?



$F_0$  is proportional to the wave amplitude. To make  $\overline{P}_m$  large (for a given wave) the phase angle  $\varphi$  should be small (resonance), since  $(\cos \varphi) = 1$  for  $\varphi = 0$ . Resonance, or if not resonance use control system to obtain small  $\varphi$ .

In this simple example, at optimum radiated-wave generation, the maximum absorbed energy equals 100 percent of the incident wave energy. Note also that the required, optimum, radiated wave has the same amplitude as the incident wave. Thus,

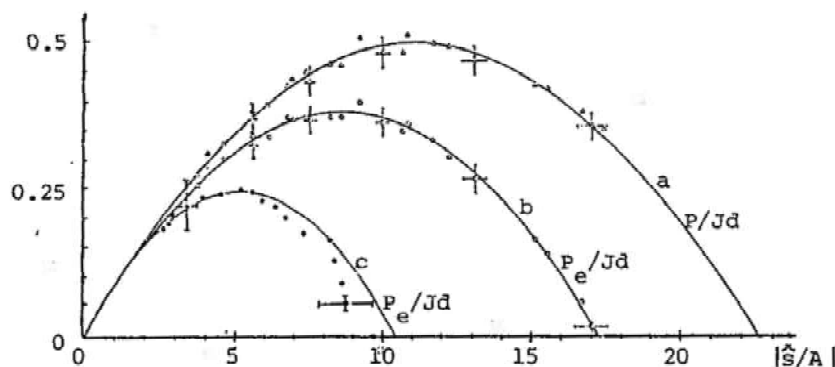
$$P_{r,OPT} = P_{a,MAX}$$

Observe that, in order to absorb, from the sea, the theoretically maximum wave power, **it is necessary that the wave-absorbing oscillating system, at optimum, has an ability to radiate as much power as the theoretically maximum absorbed power.**

This statement is valid also for systems of different geometrical configurations, where the maximum absorbed power is less than 100 percent of the incident wave power, provided **the required optimum oscillation can be realised**, that is, when no physical amplitude limitation, or other constraint, prevents the desired radiated wave from being realised.

$s$  = heave amplitude  
 $A$  = amplitude of incident wave  
 $Jd$  = incident wave power

$P$  = wave power absorbed by resonant buoy [curve a]  
 $P_e$  = power converted to electricity by resonant buoy [curve b]  
 $P_e$  = power converted to electricity by latching-controlled buoy [curve c]



Budal, K., Falnes, J., Kyllingstad, Å. and Olstedal, G.: "Experiments with point absorbers". Proceedings of First Symposium on Wave Energy Utilization, Gothenburg, Sweden, pp 253-282, 1979.

**Resonant heaving body B in wave channel  
with wavemaker W and beach A**

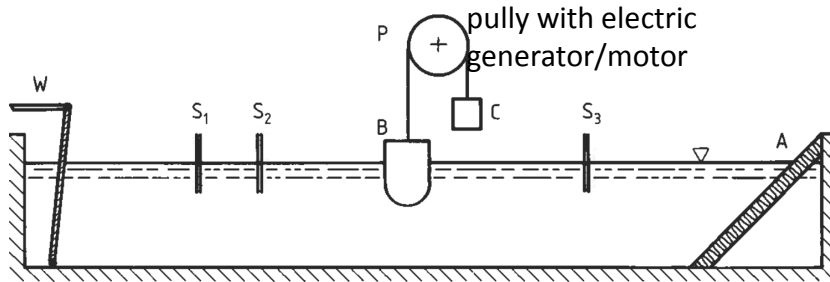
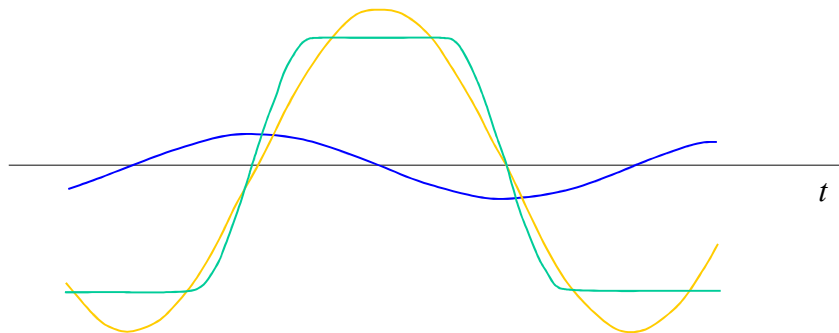


Illustration from Falnes, J. and Budal, K (1978): "Wave power conversion by point absorbers". Norwegian Maritime Research, Vol 6, No 4, pp 2-11.

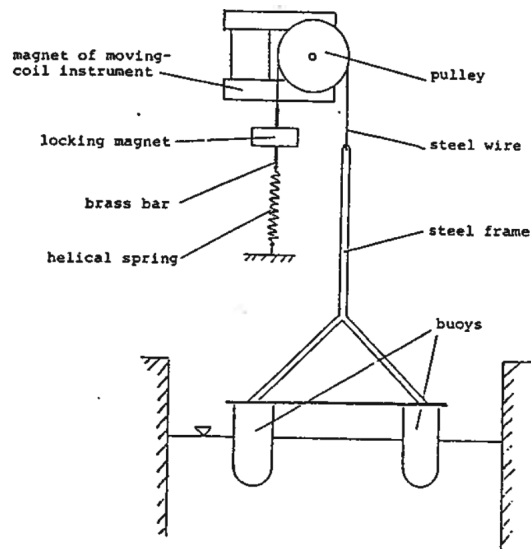
*Fig. D.18. Experimental arrangement for study of heaving body (B) in wave channel. The channel comprises a wave maker (W) and an artificial beach (A). The body (B) is suspended on a flexible wire which runs over a pulley (P) to an adjustable counterweight (C). The wave is measured by means of resistive two-wire probes ( $S_1, S_2, S_3$ ).*



Optimal phase at resonance

Phase control by latching

Laboratory arrangement for latching-controlled wave-power buoy in wave channel



Budal, K., Falnes, J., Kyllingstad, Å. and Olstedal, G.: "Experiments with point absorbers". Proceedings of First Symposium on Wave Energy Utilization, Gothenburg, Sweden, pp 253-282, 1979.

In this simple example, at optimum radiated-wave generation, the maximum absorbed energy equals 100 percent of the incident wave energy. Note also that the required, optimum, radiated wave has the same amplitude as the incident wave. Thus,

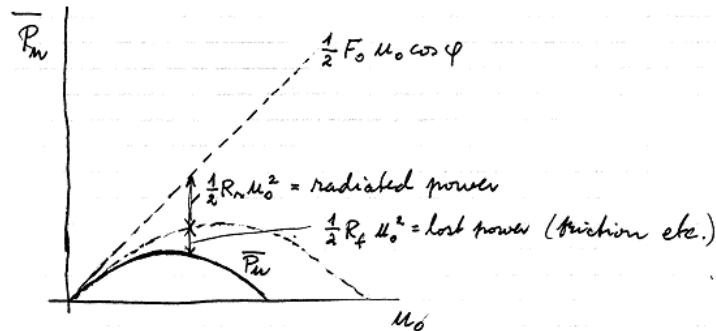
$$P_{r,OPT} = P_{a,MAX}$$

Observe that, in order to absorb, from the sea, the theoretically maximum wave power, **it is necessary that the wave-absorbing oscillating system, at optimum, has an ability to radiate as much power as the theoretically maximum absorbed power.**

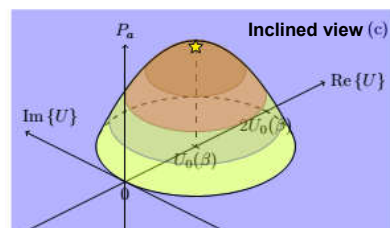
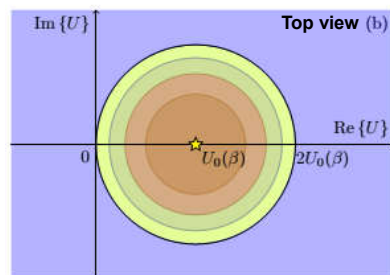
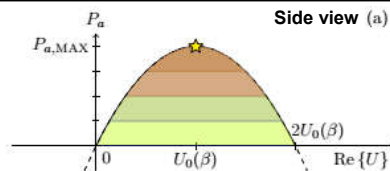
This statement is valid also for systems of different geometrical configurations, where the maximum absorbed power is less than 100 percent of the incident wave power, provided **the required optimum oscillation can be realised**, that is, when no physical amplitude limitation, or other constraint, prevents the desired radiated wave from being realised.

$$\overline{P}_m = \frac{1}{2} F_0 u_0 \cos \varphi - \frac{1}{2} R_r u_0^2 - \frac{1}{2} R_f u_0^2$$

How does  $\overline{P}_m$  depend on oscillation amplitude  $u_0$ ?



$F_0$  is proportional to the wave amplitude. To make  $\overline{P}_m$  large (for a given wave) the phase angle  $\varphi$  should be small (resonance), since  $(\cos \varphi)_{\max} = 1$  for  $\varphi = 0$ . Resonance, or if not resonance use control system to obtain small  $\varphi$ .



### The wave-power "island"

illustrates the real-valued absorbed wave power  $P_a$  versus a complex oscillation amplitude  $U$ , where  $|U|^2 = U U^*$  equals the radiated power  $P_r$ . The phase of  $U$  is chosen in order to make  $U$  real and positive when it has the same phase as the excitation force from the incident wave. The optimum value  $U_0$  is a positive real quantity.

$$P_{a,MAX} = P_{r,OPT} = |U_0|^2$$

$$P_{a,MAX} - P_a = |U_0 - U|^2$$

These simple equations are applicable to many different types of wave-energy converters (WECs). Assuming that the power take-off (PTO) machinery is equipped with sufficient control, we may consider  $U$  to be an independent complex variable. The optimum value  $U_0$  is, however, proportional to the incident wave amplitude  $A$ , and, moreover, it is a function of  $\beta$ , the angle of wave incidence.

Figures from Falnes & Kurriawan, 2015, R. Soc. open sci. 2: 140305. <http://dx.doi.org/10.1098/rsos.140305>

## Three wave-power inventor pioneers



**Yoshio Masuda** (1925 – 2009)

Started already in 1947 with experiments to test devices for utilising wave energy in Japan.



**Stephen Salter** (1938 – )

started 1973 wave-power research at the University of Edinburgh, Scotland.



**Kjell Budal** (1933 – 1989)

initiated in 1973 wave-power research at NTH (part of pre-NTNU university), Trondheim, Norway.

**80 m long vessel Kaimei (= sea light) for testing various types of wave-activated air turbines. (Japan, late 1970s and early 1980s)**

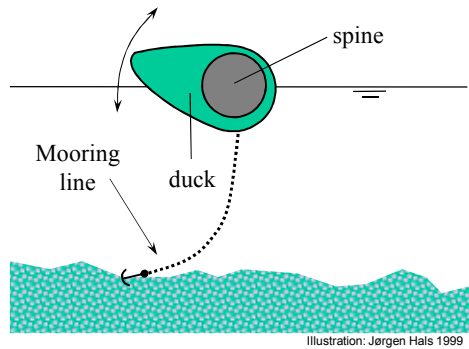


Copyright: JAMSTEC, Japan



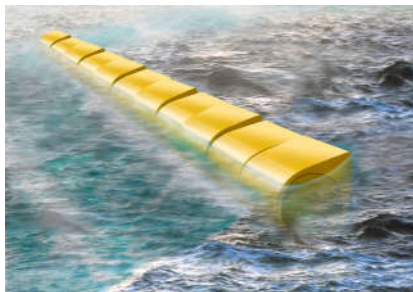
## The Salter duck

- In 1974 Stephen Salter published a paper on a device which has become known as the “Salter duck”, the “Edinburgh duck” or simply the “Duck”, because the device, in its pitching oscillation, resembles a nodding duck. Several ducks share a common spine. The relative pitch motion between each duck and the spine is utilised for pumping hydraulic fluid through a motor.

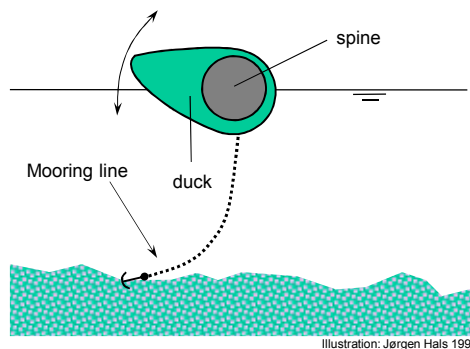


## Salter's nodding Duck

Scotland (Stephen Salter, University of Edinburgh)



Energy conversion through pumps, pressure tank, hydraulic motor an electric generator



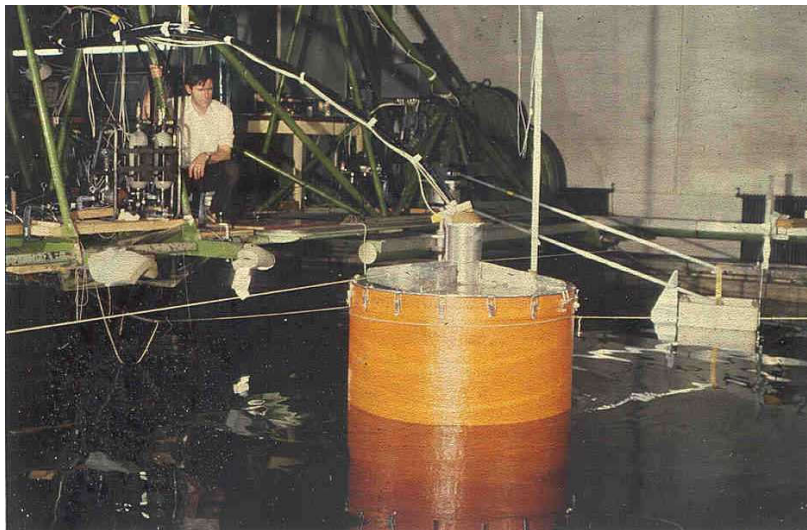
Research work in Edinburgh with the spine, a long tube, at least 100 m long, has later evolved into the Pelamis project:

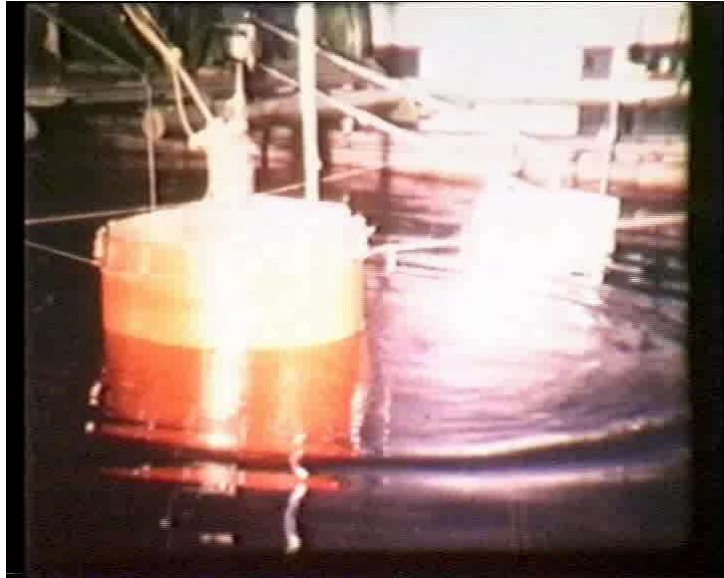


Video clip of "Pelamis"

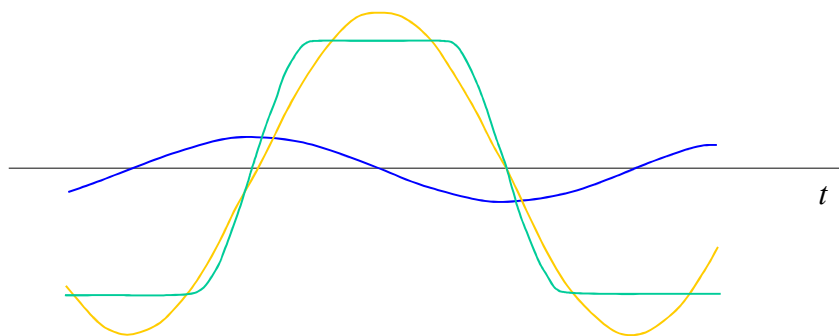
PELAMIS WAVE POWER LTD <<http://www.pelamiswave.com/galleryvideo.php>>

**Kjell Budal with his phase-controlled power-buoy model (type E) in the Trondheim towing tank 1978**





Phase-controlled power-buoy model (type E) under test in Skipsmodelltanken, Trondheim, 1978. Video clip [also on [http://folk.ntnu.no/falnes/w\\_e/](http://folk.ntnu.no/falnes/w_e/).]



Optimal phase at resonance

Phase control by latching

## Array of point absorbers

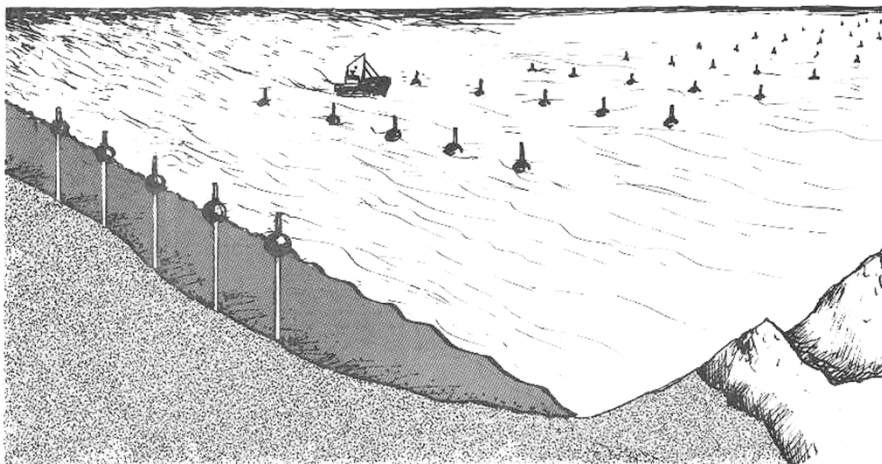
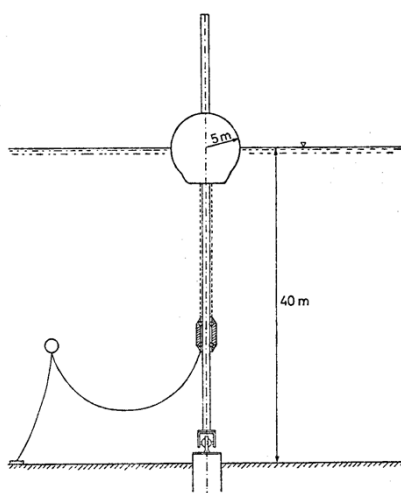


Figure from Stortingsmelding [White paper] nr. 65 (1981-82): *Om nye fornybare energikilder i Norge* [On new renewable energy sources in Norway]. The Royal Ministry of Petroleum and Energy, Oslo, 1982

## The Trondheim point absorber



Source: K. Budal, 1981



Photo: J. Falnes, 1983

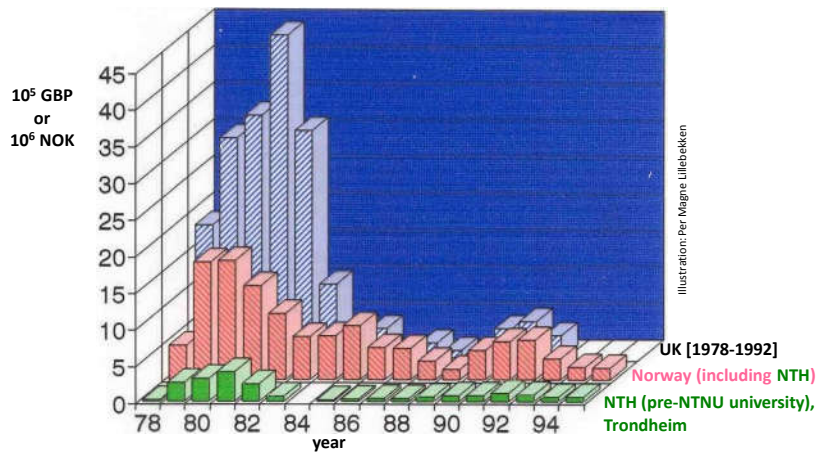


Budal's phase-controlled power buoy model (type N2) at the test site in Trondheimsfjorden.



Phase-controlled power-buoy model (type E) under test in the Trondheim Fjord, 1983. Video clip [also on [http://folk.ntnu.no/falnes/w\\_e/](http://folk.ntnu.no/falnes/w_e/).]

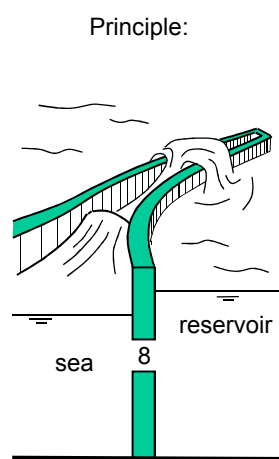
## Governmental funding of wave-power R&D in Norway and in the UK



During the early 1980s, when research teams were ready to test models the real sea, increased funding was needed. In stead conservative governments in the UK and in Norway reduced funding of wave energy.

## The tapered channel

- The tapered channel is a horizontal channel which is wide towards the sea where the waves enter and gradually narrows in a reservoir at the other end. As the waves pass through the channel, water is lifted over the channel wall and into the reservoir due to the shortage of space which occurs as the channel gets narrower.



Norwave's tapered-channel WEC (350 kW) at Toftøy [40 km NW from Bergen], Øygarden, Norway.

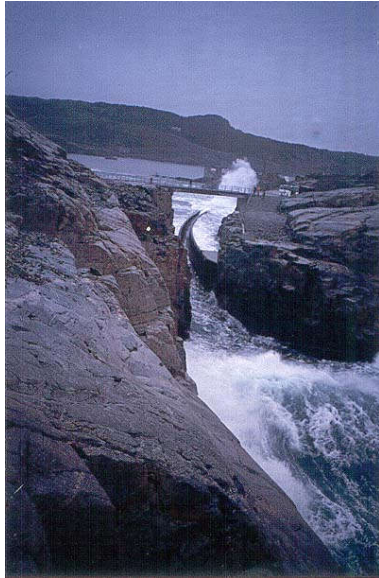


Photo: NORWAVE AS, 1986



Photo: NORWAVE AS, 1986

OED (Ministry of Petroleum and Energy) issued 1987 two reports on NORWAVE's and Kvaerner's wave-power prototypes, 40 km off Bergen. One report, "*Norwegian wave power plants 1987*", with text in Norwegian and English, was open.

The other report, "*Bølgekraftverk Toftestallen: Prosjektomiteens sluttrapport 31.12.1987*", had only closed distribution. It contained more detailed information, in the Norwegian language, only.

## NORSKE BØLGEKRAFTVERK 1987

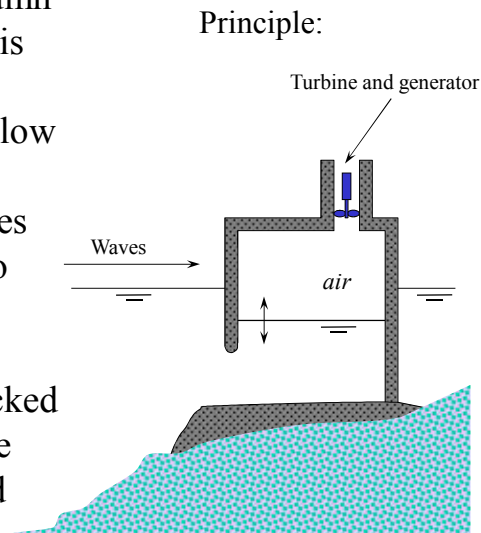


## NORWEGIAN WAVE POWER PLANTS 1987

OED  
MPE

## Oscillating water column (OWC)

- In an oscillating water column a part of the ocean surface is trapped inside a chamber which is open to the sea below the water line. When the internal water surface moves up and down in response to incident waves outside the chamber, the air in the chamber is pressed and sucked through a turbine due to the generated overpressure and underpressure.

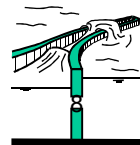
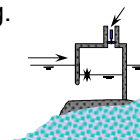


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The other report, "*Bølgekraftverk Toftestallen: Prosjektkomiteens sluttrapport 31.12.1987*", had only closed distribution. It contained more detailed information, in the Norwegian language, only.

By end of 1988 Kvaerner's 500 kW OWC prototype had delivered 29 MWh to the local utility Nordhordland Kraftlag.

It seems that the installed power capacity was much too large!



By end of 1991 NORWAVE's 350 kW TapChan prototype had delivered 691 MWh to the local utility Nordhordland Kraftlag.

Energy deliveries as informed by Nordhordland Kraftlag in letter 1993



In the early 1980s Kværner Brug AS planned a multi-resonant OWC WEC standing on 25 m deep sea bed.

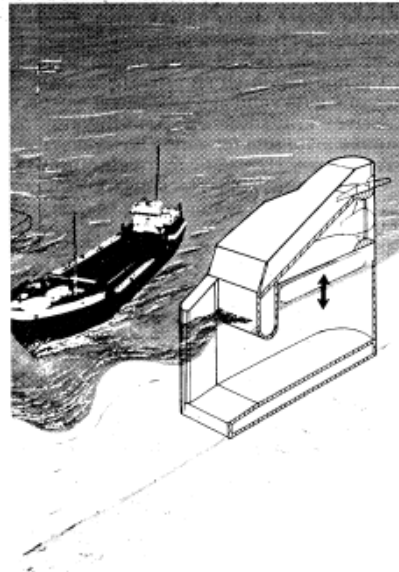


Figure from Stortingsmelding [White Paper] nr. 65 (1981-82): *Om nye fornybare energikilder i Norge* [On new renewable energy sources in Norway].

Fig. 7. Kværners svingende vannsøyle er bygget i betong. Vannsøylen inne i konstruksjonen settes i bevegelse (pil) og driver en luft-turbin bakerst til høyre.

Kværner Brug's 500 kW WEC of the OWC type in a very steep cliff on island Toftøy, 40 km NW from Bergen. The red part, below the generator housing, is the housing for a self-rectifying air turbine.

Constructed during 1985 and destroyed by a storm during the last week of 1988.



Foto: J. Falnes 1985

Array of Pelamis WEC units



Drawing copied 2016-09-13 from <http://nmrec.oregonstate.edu/pelamis-attenuator>

Array of point-absorber WEC units

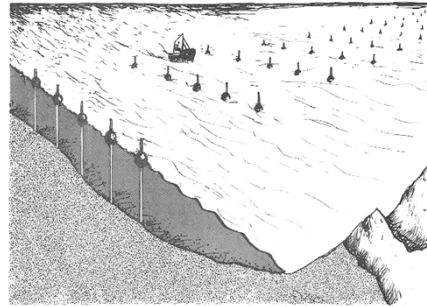
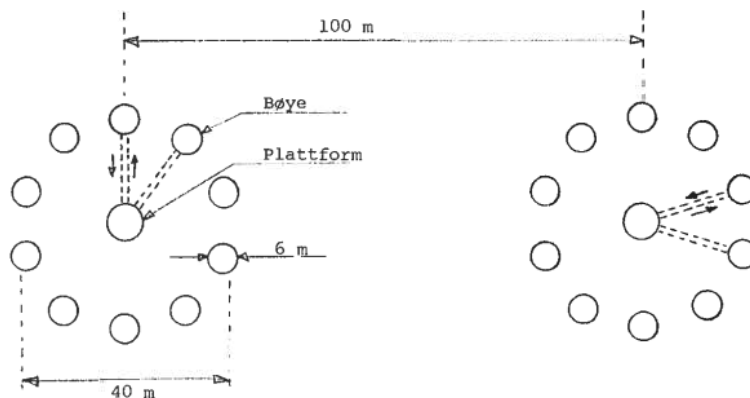


Figure from: St.meld. nr. 65 (1981-82): *Om nye fornybare energikilder i Norge*, Oslo, 1982.

The central part of each Pelamis cylinder contributes less to the needed wave generation than the two end parts. But, unfortunately, it contributes fully to the extreme structural and mooring forces.

Point absorbers may require more sophisticated technology to be developed. Each PA unit may, through a flexible hose, deliver primary-converted hydraulic energy to a common hub unit, which contains a hydraulic motor (or turbine) and an electric generator.

**Ten point-absorber WEC units, each 200 kW, shearing a common platform hub with 2 MW hydraulic motor and electric generator.**

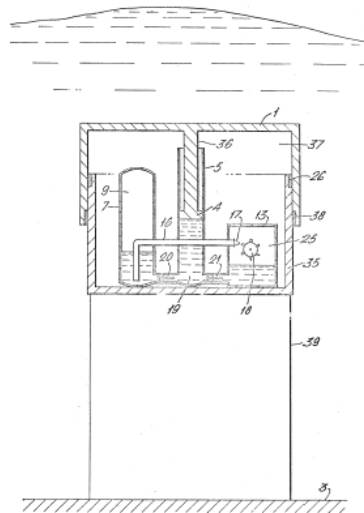
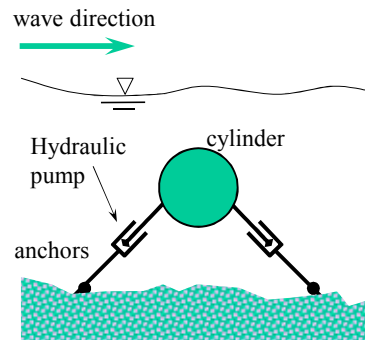


Proposal and drawing by Kjell Budal 1978 [cf. *Preliminary design and model test of a wave-power converter: Budal's 1978 design Type E*. Technical report, Institutt for fysikk, NTH, Trondheim, 1993.]

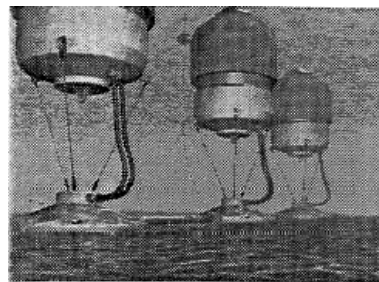
# The Bristol cylinder

- This wave energy device was proposed by David Evans at the University of Bristol in England. In response to an incident wave the submerged horizontal cylinder oscillates vertically and horizontally. With a sinusoidal wave the combined oscillation results simply in a circular motion whereby all the incident wave energy may be absorbed provided the hydraulic power take-off is able to provide for optimum amplitude and optimum phase of the circular motion. The hydraulic power take-off is built into the anchors.

Principle:



**A phase-controlled submerged pulsating-volume device with hydraulic power take-off**  
(Budal patent. Application filed 1977)



**Artist impression of a cluster of AWS devices**

(in 3EWEC-1998 paper by Rademakers, van Schie, Schuitema, Vriesema and Gardner )

“[Archimedes Installation at EMEC Next Year](#)”, Maritime Journal, 30 July 2008.

# AWS

## “Archimedes Wave Swing”

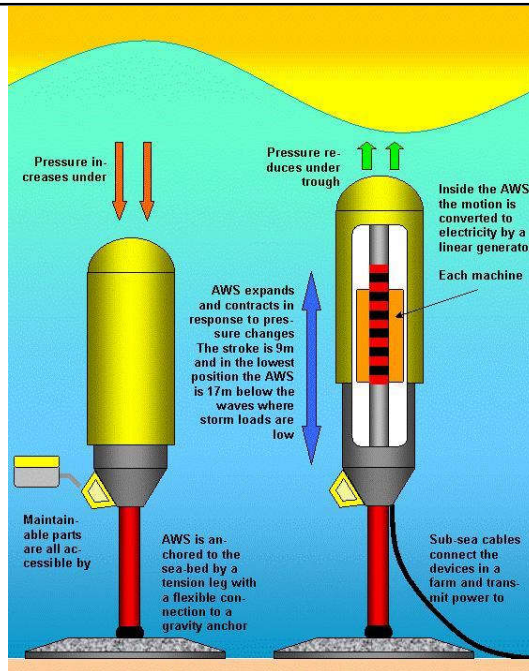
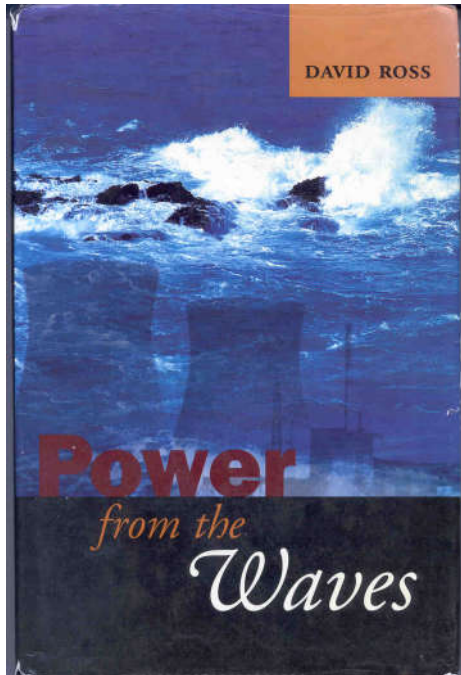


Illustration in the paper “[Archimedes Installation at EMEC Next Year](#)”, Maritime Journal, 30 July 2008.

[http://www.maritimejournal.com/archive101/2007/september/renewables/archimedes\\_installation\\_at\\_emec\\_next\\_year](http://www.maritimejournal.com/archive101/2007/september/renewables/archimedes_installation_at_emec_next_year)



David Ross: "*Power from the Waves*", (Oxford University Press, 1995) (ISBN 0-19-856511-9)

An easy-read book written by the British free-lance journalist David Ross.

David Ross, in his 1995 book "Power from the Waves" reports (p.180) from a wave-energy meeting in Brussels 1991:



Stephen Salter

The discussion saw another round in the debate - - - about whether it was best to go to sea sooner or later. Professor Salter insisted:

I don't want to be the first wave power device at sea. I want to be the last one. I want to make all the mistakes in private, with instruments to tell me what mistakes I have made so that I don't do it again. I want to do all the difficult things in the laboratory. There was enthusiasm for air ships, but the R101 crashed. Airships finally died when the Hindenburg died. If you had a spectacular disaster with one wave energy device, you could drag everything down, too.

### **Recommendations:**

To make large-scale utilisation of ocean-wave energy a future reality,  
I recommend a 3-step development program as follows:

Establish international agreements concerning ownership of the energy that ocean waves may transport, possibly thousands of kilometres, **across offshore national territorial borders.**

R&D&D programmes for various kinds of single wave-energy conversion (WEC) units of power take-off (PTO) capacity in the range of **100-300 kW.**

When such WEC units, deployed in the sea, have demonstrated an annual energy production equal to the PTO's power capacity multiplied by at least 2500 hours, they may become candidates for a R&D&D programme on wave power plants consisting of a huge number of **mass-produced** cooperating WEC units.

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## THE END

## SLUTT



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