Wind Power – Part I

John C. Bean

Outline

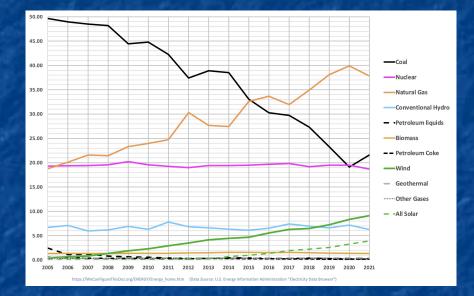
Wind's variation with locale, time & altitude Wind's energy and power Implications for all wind turbine designs / Implications for wind farm location & layout Aero 101: DRAG (exploited in Savonius vertical axis wind turbine - VAWT) LIFT (exploited in Danish horizontal axis wind turbine - HAWT & Darrieus VAWT) How their use of lift & drag explain and limit performance of these turbines Aero 201: Bernoulli's Equation / Betz's Limit of Lift + Drag turbines / Limit on pure Drag Turbines The Danish Experience: Anyone can make a working wind turbine, problem's KEEPING it working! Aerospace failures vs. the farm machinery company now supplying the world with turbines: Shared & verified performance data driving use of robust & standardized components The hard-luck lessons about failsafe turbine over-speed protection Leading to the Danish turbine's current supremacy and ongoing trends in its deployment

(Written / Revised: August 2022)

Wind Power

Wind is now the #3 low-greenhouse-gas source of U.S. power (at 9.1%)¹

It's behind nuclear (18.6%) & hydro (6.3%), but still well ahead of solar (3.9%)



The government now identifies it as one of the three cheapest forms of U.S. power ²
 With certain natural gas plants & geothermal plants sharing that distinction
 Industry sources instead identify it as the single cheapest form of U.S. power ²
 The result: Over half of newly built U.S. power generation capacity is due to wind ³
 1) From my web note set: U.S. Energy Production and Consumption (pptx / pdf / key)
 2) From my web note set: Power Plant Economics: Analysis Techniques & Data (pptx / pdf / key)

3) U.S. Federal Energy Regulatory Commission: Office of Energy Projects – Energy Infrastructure Update – May 2018

And there seem to be so very **many** choices of wind turbine:



"Danish"



Simple Darrieus



Eggbeater Darrieus



Spiral Darrieus



Simple Savonius



Spiral Savonius



Photo credits,,clockwise from top left:

http://www.christopherteh.com/blog/2010/11/wind-energy/ https://www.researchgate.net/figure/Quiet-Revolution-twisted-Darrieus-wind-turbine_fig1_252509085 https://en.wind-turbine-models.com/turbines/93-dornier-darrieus-55 http://www.power.news/2017-08-15-vertical-axis-wind-turbines-harvest-turbulent-wind-more-efficiently.html https://mechaniclove.com/wind-turbine-types/main-qimg-94635f1f0ea3aa06995be51f04c4e43a-c/ https://www.pinterest.com/mcdegen/cm_urban-scene/ https://www.decoupagedesignsusa.com/dutch-windmill-decoupage-napkin/

But wind power is not without controversy, including: Heated arguments about Wind Power's impact upon flying creatures And about its impact on human neighbors Including noise, radio/TV interference, disruption of natural vistas, yielding: NIMBY = Not in MY backyard (!!#@\$!) Further, there is huge disagreement about the best way to implement Wind Power: The public and press are fascinated by unique & compact turbine designs Such as the non-Danish turbines pictured on the preceding page But Danish turbines now provide virtually all worldwide Wind Power Further, the industry is obsessed with ever bigger (more intrusive) versions! Finally, answers seem buried in aerodynamic theory so opaque and obscure that aerodynamicists seldom even TRY to explain it to anyone but aerodynamicists!

An Introduction to Sustainable Energy Systems: WeCanFigureThisOut.org/ENERGY/Energy_home.htm

Making this note set one of my most difficult to research & write Which rather surprised me, given that it started out looking so easy Especially given all of our relevant personal experience with things such as Childhood pinwheels: And paper airplanes:





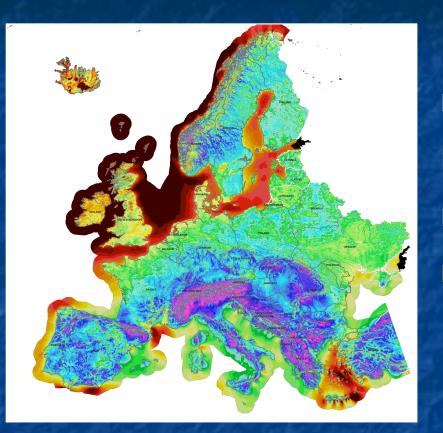
But in the spirit of WeCanFigureThisOut.Org, I believe I have now figured it out And in this note set I'm eager to share with you what I have learned Starting at the **very** beginning with what I have learned about:

> Left: http://living.thebump.com/craft-paper-pinwheel-young-children-15846.html Right: http://www.pbs.org/parents/education/science/activities/first-second-grade/everyday-science/

WIND

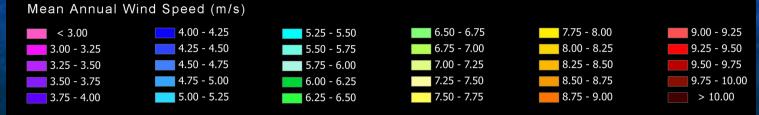
An Introduction to Sustainable Energy Systems: WeCanFigureThisOut.org/ENERGY/Energy_home.htm

Yearly averaged wind speeds for Europe (at 80 meters):



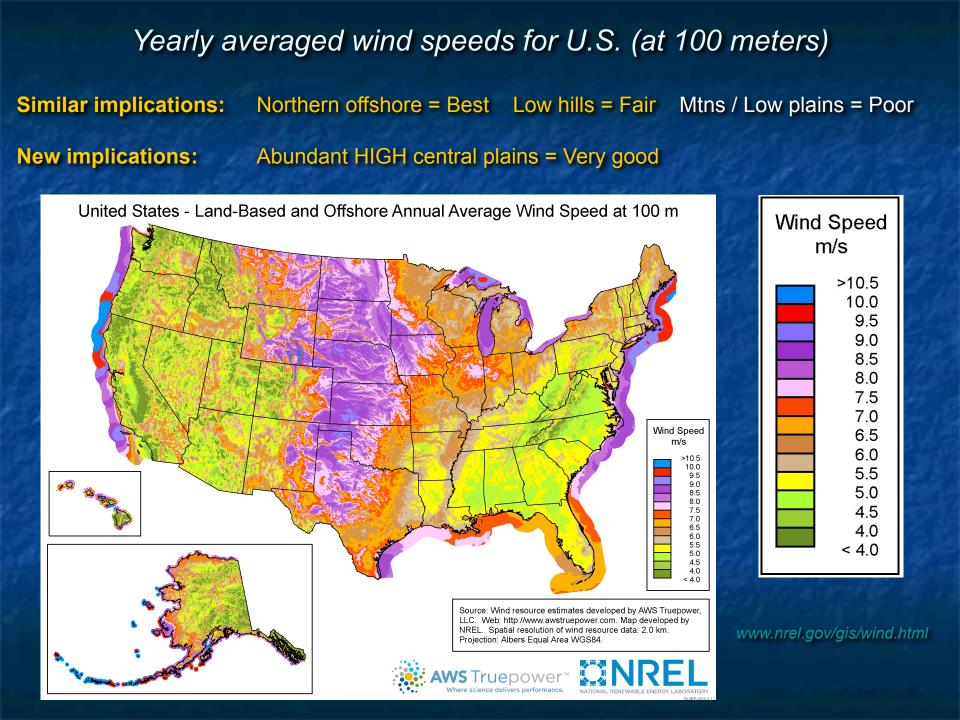
The implications for wind power? BAD = High mountains (wind blocked/lifted) FAIR = Low hills POOR = Low altitude plains (e.g. the Veneto) BEST = Offshore Especially at northern latitudes!

(FYI: 1 m/s ~ 2 ¼ mph)



This map depicts the approximate annual average wind speed at 200 meter resolution and an 80 meter hub height. It was created by AWS Truepower using

Map & its key were extracted from: https://aws-dewi.ul.com/assets/Wind-Resource-Map-EUROPE-11x17.pdf

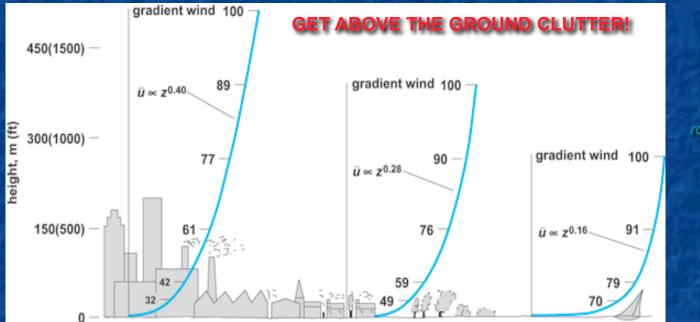


But why did those maps carefully note the height above ground?

Because wind speed increases rapidly with height

Due to things near the ground impeding air flow (plants, trees, buildings . . .)

(Ask any ant: While rain's a big concern, ground level wind is no problem!)



http:// rockets2sprocket s.com/issuecross-windswind-tunnels/

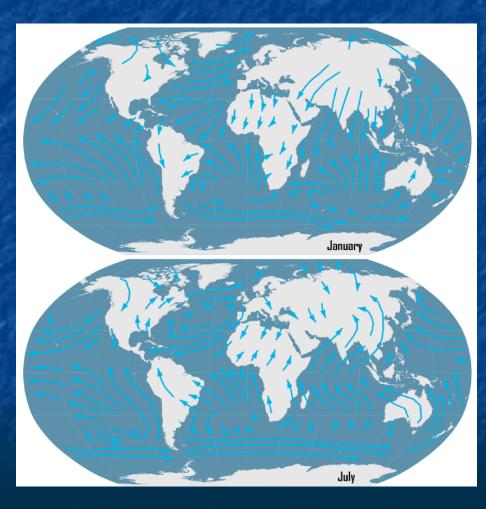
This variation, known as **WIND SHEAR**, can be modeled as:

Wind speed α (height) ⁿ with n ~ 1/4 to 1/2

WHEN do we get the strongest winds?

You've probably noticed seasonal variations in wind direction

Illustrated here are typical shifts in worldwide January vs. July wind direction:

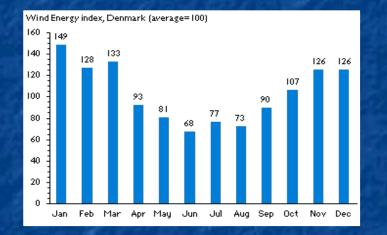


https://transportgeography.org/?page_id=386

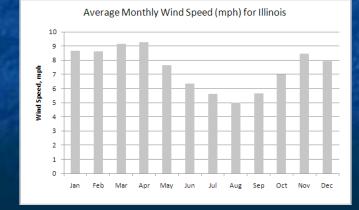
Wind speed also follows an ANNUAL CYCLE

WindPower.Org generated this plot of Danish winds falling over summer months,

a trend they identified as typical for "temperate" areas of the world: 1



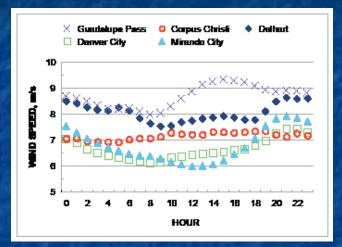
Consistent with that claim, I found this plot of annual U.S. (Illinois) wind speeds: ²



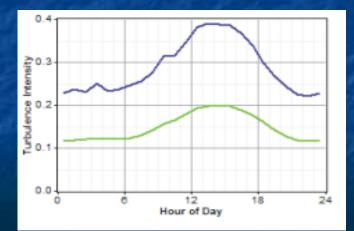
1) https://transportgeography.org/?page_id=386 2) https://www.isws.illinois.edu/statecli/wind/wind.htm

Wind speed is also subject to strong DAILY CYCLES:

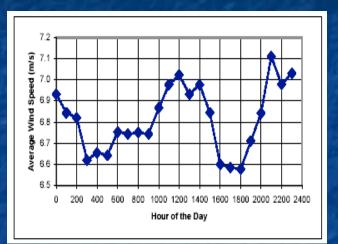
With that speed peaking in the late afternoon in many (but not all!) locales



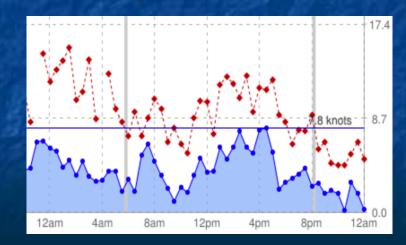
TEXAS: www.seco.cpa.state.tx.us/publications/ renewenergy/windenergy.php



NETHERLANDS: www.ekopower.nl/wind-energymonitoring-data-logger-data-processing.htm



ONTARIO CANADA: www.omafra.gov.on.ca/english/ engineer/facts/03-047.htm



WISCONSIN: www.windpowerweather.com/history?date=last2days

But what **POWER** do such winds offer?

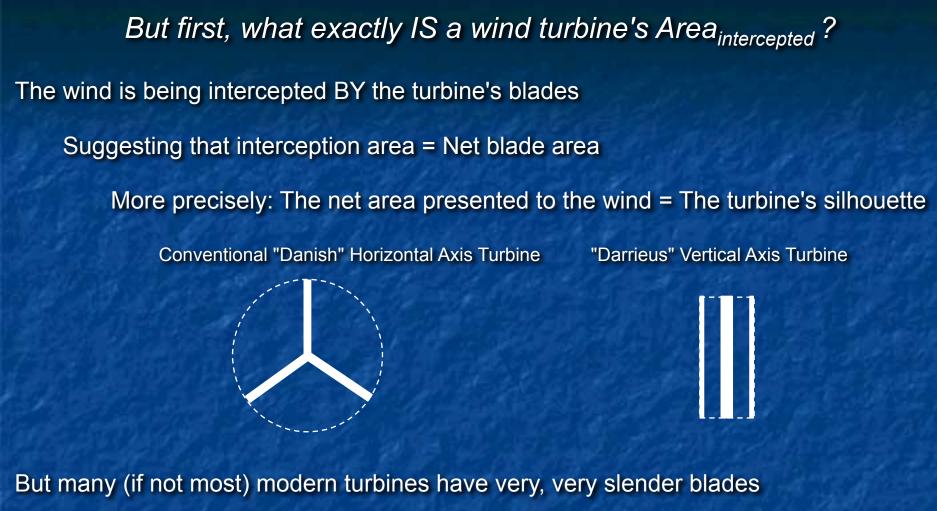
Turbine generator (with whirly things)



The **kinetic energy** per volume of wind = $\frac{1}{2}$ (M / volume) $v^2 = \frac{1}{2} \rho v^2$ But wind volumes arrive at the turbine at a rate proportional to the wind's velocity So the power delivered = ($\frac{1}{2} \rho v^2$) v Area_{intercepted} = $\frac{1}{2} \rho v^3$ Area_{intercepted} OR:

Power of Wind Flow / Area intercepted = $\frac{1}{2} \rho v^3$

That v³ dependence has far reaching consequences!



Suggesting that their "intercepted wind areas" might be disturbingly small, which would radically decrease the wind power harvested by such turbines!

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But as ships have "bow waves," so do the blades of wind turbines



https://www.popsci.com/navyagrees-to-limit-activities-incalifornia-and-hawaii-to-savewhales

And, as demonstrated by these happily surfing dolphins

these pressure waves spread surprisingly far forward and to the sides

The air flow affected by a turbine blade

is thus significantly wider than that blade's physical width

THEN: If the blades turn fast enough, wind caught up in one blade's "bow wave" cannot pass completely through that bow wave before it's caught in the bow wave of the **next** blade sweeping by

To the wind, turbines thus appear almost as big as their swept area That is: Map out the 3D volume swept out by the rotating turbine Then take its full 2D cross section as viewed by the wind The result: Instead of these these blade silhouette cross sections:

Conventional "Danish" Horizontal Axis Turbine

"Darrieus" Vertical Axis Turbine





The wind flow Area_{intercepted} for these turbines is more like this:

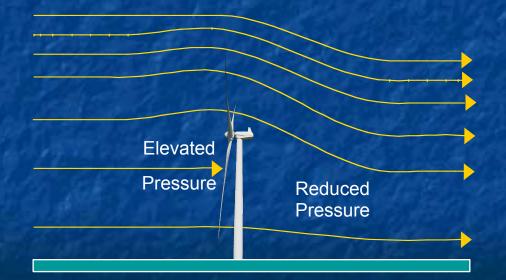


And these much larger areas determine their ability to harvest wind power

"Far reaching consequences" of wind's velocity cubed power dependence Power of Wind Flow / Area intercepted = $\frac{1}{2} \rho v^3$

That wind power is due to the air's movement To remove all of that power FROM the wind, and transfer it to our turbine, we would have to STOP that wind But if our turbine completely stopped the air flow, no more wind could then reach it! So as a compromise, what if our turbine just slowed the wind by $\frac{1}{2}$? That sounds like it would allow us to extract only $\frac{1}{2}$ of the wind's power, right? **NO!** Thanks to that v^3 , we might extract close to 7/8's of the wind's power: Entering wind: $P_{in} = \frac{1}{2} \rho v_{in}^3 A$ Exiting wind: $P_{out} = \frac{1}{2} \rho (v_{in}/2)^3 A$ $P_{extracted} = P_{in} - P_{out} = \frac{1}{2} \rho v_{in}^3 (1-1/8) A = 7/8 P_{in}$ That's good news. The bad news is that we'd still slow flow down by 50%

Suggesting that we might be forced to use just ONE ROW of turbines Because a 2nd row (behind the 1st row) would get so much slower wind! But we are now saved by those higher speed winds ABOVE the turbine And there is a **whole lot** of higher speed wind above it! The turbine affects airflow across its ~ **full** rotational area Air flowing **through** the turbine area is radically slowed Which lowers air pressure behind the turbine **Which sucks faster-flowing air down from above:**

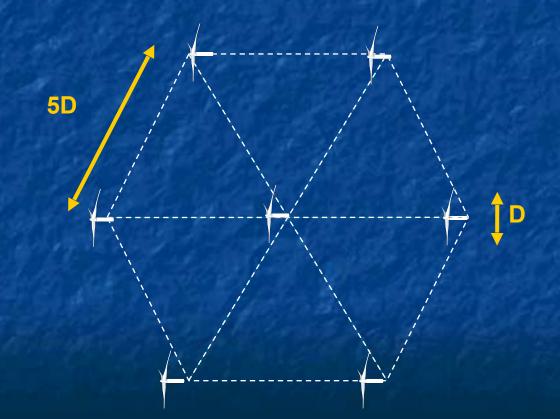


To the turbine's SIDES there is a similar backfill by faster-higher-altitude airflow

Drawn-in higher-altitude wind can thus regenerate lower-altitude wind

At least if it is given enough time (=> space behind & beside the turbine) to do so To allow for that regeneration time/space:

Wind farms generally separate their turbines by ~ five blade diameters

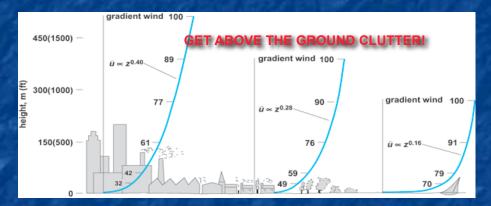


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But how big should **D** (turbine diameter) & **5D** (turbine spacing) be?

Based on the v³ dependence, mild increases in speed yield major increases in power

And the easiest way to access speedier winds, is to just build a taller tower



A taller tower **ALSO** accommodates a larger turbine => larger Area_{intercepted}

MAKE THINGS AS LARGE AS POSSIBLE! => Huge, widely spaced turbines:

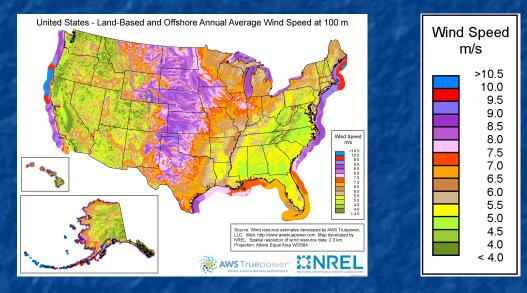


And where should these tall/huge widely spaced turbines be built?

Based again upon wind power's v³ dependence:

Slightly higher wind speed => Hugely higher wind power

Which, combined with Wind Resource maps such as this:



Suggests this ranking of possible wind farm locations:

BEST:	Off northern coasts
GOOD:	On high plains
POOR:	In mountains & low plains

And, of course, we should look for locations with steady winds **NOT NECESSARILY!**

From: Power / Area_{intercepted} = $\frac{1}{2} \rho v^3$ Using a density for air: $\rho_{air} = 1.2 \text{ g} / \text{liter}$ We'd calculate these wind powers: Wind Speed Wind Power 4.8 watts / m² x A_{intercepted} 2 m/s (4.4 mph) 38.4 watts / m² x A_{intercepted} 4 m/s (8.8 mph) 307 watts / m² x A_{intercepted} 8 m/s (17.6 mph) 2457 watts / m² x A_{intercepted} 16 m/s (35.2 mph)

Note the HUGE differences in power!

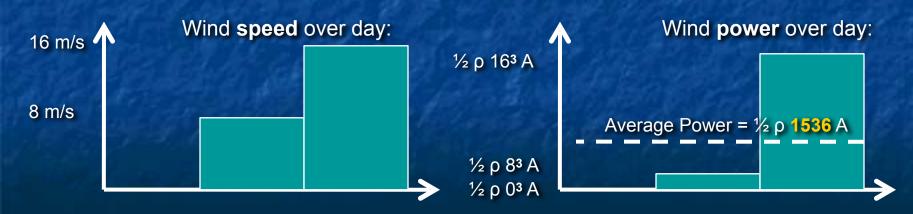
Then consider two hypothetical locales, both with **average wind speed of 8 m/s**: At the 1st locale wind speed is constant throughout the day at 8 m/s But at the 2nd locale: 0 wind for 8 hrs, 8 m/s for next 8 hrs, 16 m/s for final 8 hrs Compare the accumulated wind power (= Energy) for both locales

(Using formula from above for two cases with SAME average wind speed of 8 m/s)

LOCALE 1: Constant daily wind speed of 8 m/s:



LOCALE 2: Variable wind speed averaging (over day) 8 m/s:



But is my simplified variable wind scenario realistic? My model suggests that intermittent wind might **TRIPLE** daily wind power 307 watts / m² (for steady 8 m/s) => 921 watts / m² = (0 + 307 + 2457) / 3 What does REAL DATA for REALISTICALLY VARIABLE WIND LOCATIONS give? Based on real data + modeling, an earlier edition of NREL's U.S. map 1

translated wind speed into wind power as follows:

d
beed
0
5
0
8
.1

For 8 m/s average wind speed locations, NREL cited powers of 600 watts / m² Thus, according to NREL, real variable winds ~ DOUBLE daily wind power Suggesting real winds are high for ~ 1⁄4 of day (vs. my assumed 1/3 of day) 1) https://wecanfigurethisout.org/ENERGY/Web_notes/Wind/Wind%20-%20Supporting%20-%20Files/ Wind%20Resourses%20US%20-%20NREL%20-%202009.jpg Looking back upon what we just learned about wind:

You (or certainly I) might have thought that an ideal wind farm should consist of: Lot's of compact closely-spaced turbines located atop nearby ridgelines or packed into nearby plains, at least if those ridgelines & plains offered strong steady winds But then a little science taught us that wind power increases as its speed cubed That, plus mapping of real-life winds by locale, time and height, taught us that ideal wind farms should instead consist of: Huge well separated turbines located off our northern coasts or across our high western/midwestern plains which offer high peak (but not necessarily steady) winds

AERODYNAMICS 101

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How can we best capture the wind's energy?

Which types of turbine will do the best job?



"Danish"



Simple Darrieus



Eggbeater Darrieus



Spiral Darrieus



Simple Savonius

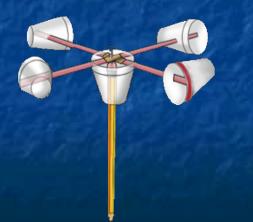


Spiral Savonius

Photo credits,,See earlier version of this slide, above

Answers are to be found in the field of **Aerodynamics** Which Wikipedia defines as: The study of the motion of air, particularly its interaction with a solid object Aerodynamics is built around two concepts, drag and lift Different wind turbine designs exploit both, but to differing degrees Drag, and drag-based wind turbines are the easiest to understand Consider a really simple drag-based wind turbine:

The DIY styrofoam cup anemometer



Its simplified top view



Left figure adapted from: https://c03.apogee.net/contentplayer/?coursetype=kids&utilityid=pseg&id=16244

Wind turns this anemometer based on unbalanced drag:

Drag (resistance to air flow) pushes ALL of the cups ? Drag on the upper and lower cups generates no torque Drag on the left cup generates strong CCW torque Drag on the right cup generates weak CW torque The combined effect? This anemometer spins CCW How fast do the individual cups end up moving? With really good bearings and really well designed cups, their speed might approach (but not exceed) that of the wind

Drag similarly propels classic "windjammers" downwind, but friction limits them to far **less** than wind speed

Picture: https://www.seacloud.com/en/yachts/sea-cloud-ii/sailing/



Air Flow

The classic "Savonius" wind turbine is just a rearranged drag anemometer:



Left: Movie showing operation of Savonius turbine made from split metal drum (link) Center: Top view revealing the close resemblance to a 2 cup anemometer Right: Commercial spiral version of a Savonius turbine: Spiral ensures that some part of one cup is ALWAYS directly facing the wind Which smoothes out the torque produced as this turbine rotates

> Left: https://www.youtube.com/watch?v=HFerHBNS9BE Center: http://verticalaxiswindturbines.blogspot.com/2010/10/how-efficient-are-vertical-axis-wind.html Right: https://www.turbosquid.com/3d-models/3ds-max-helical-savonius-wind-turbine/804711

But based on drag, Savonius turbine speed is fundamentally limited: Like our styrofoam cup anemometer, its fastest point (the outer edge of its rotor) can at best be dragged along at the wind's full speed Which introduces a key "figure of merit" for wind turbines, their: TIP SPEED RATIO (TSR) = Speed of rotor tip / Speed of Wind For ANY turbine based solely upon DRAG, Tip Speed Ratio is ≤ 1 Our windjammer is a sort of wind turbine (just being dragged along by the wind): Encumbered by a lot of water resistance, that windjammer's TSR is << 1

But for a streamlined modern sailboat, TSR should be significantly larger



TSR << 1



Left: https://www.seacloud.com/en/yachts/sea-cloud-ii/sailing/ Right: https://www.hobie.com/sail/hobie-16/

This is a plot of a modern Hobie Cat sailboat's speed vs. sailing direction: Moving down a 10 mph wind it achieves 4.5 mph: TSR = 4.5 mph / 10 mph = 0.45 But look what happens when it (& other modern sailboats) turn **away** from the wind

THEY GO FASTER THAN THE WIND: TSR = 1.4 at right angles!!

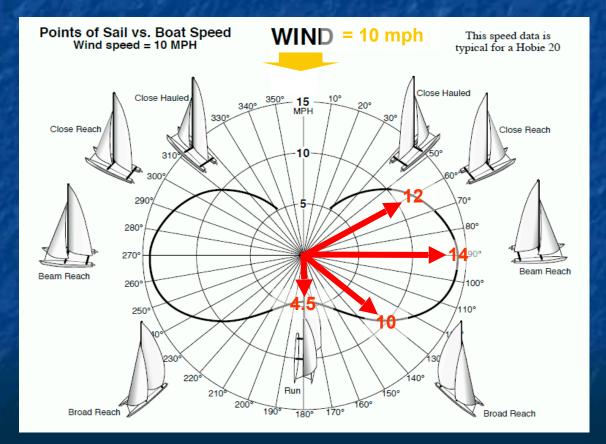


Figure = My enhanced version of: https://hobiecat16.wordpress.com/2010/09/23/points-of-sail-vs-boat-speed/

What the heck is going on? We've added aerodynamic lift Air flowing into an "airfoil" (e.g., an aircraft wing, a sailboat sail, or a turbine blade) must change its direction of movement => a change in its momentum but to alter an airflow's momentum, it must experience a force Newton's "action = reaction" demands a balancing force on the airfoil That force on the airfoil is conventionally broken down into two components: - A component parallel to the direction of the air flow = Drag - A component **perpendicular** to the direction of the airflow = Lift



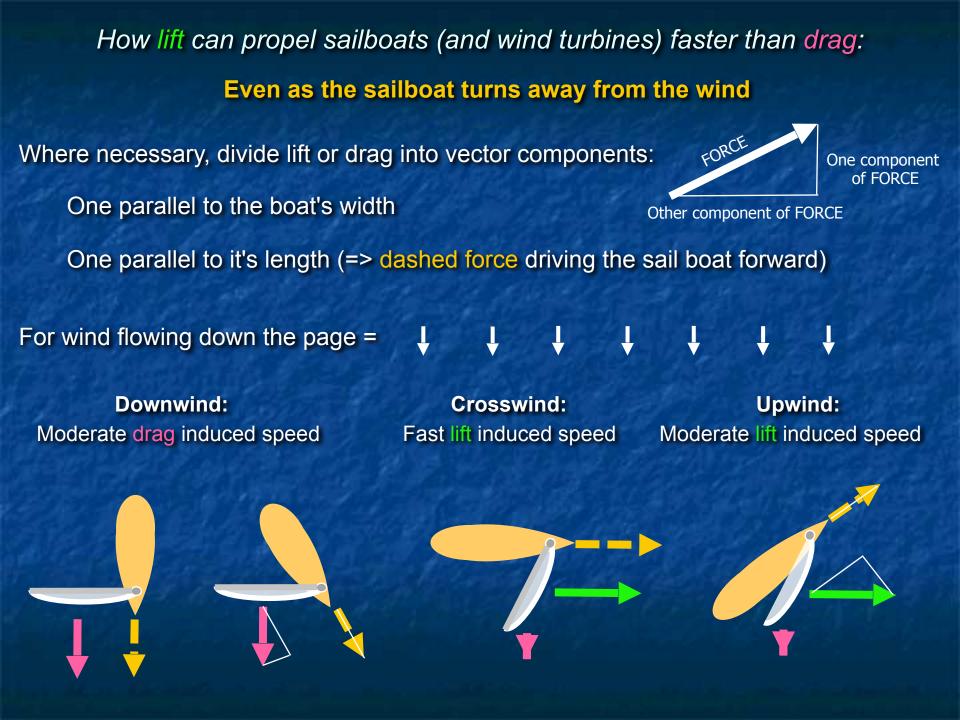
But the **tilt** of the airfoil relative to the air flow can be changed The misalignment between the two is called **Angle-of-attack** As the angle-of-attack changes, so do lift and drag But they change in different ways:

As the angle-of-attack increases from 0 to 90 degrees:

- Lift initially increases, but then falls dramatically
- Drag just increases (as the airfoil presents an increasing obstacle to air flow)



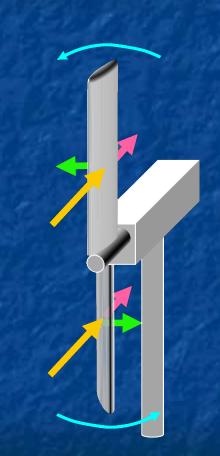
Which explains the sailboat's strange behavior - **IF** we remember that boat hulls allow easy motion forward (and backward), but sharply limit motion sideways We just have to figure out components of the sail's **lift & drag** along the hull axis:



Aerodynamic lift can similarly explain a wind turbine's rotation:

You just have to think of the blades as sailboats sailing crosswind:

Lift drives each blade sideways, The upper blade to the left The lower blade to the right Which, together, rotate this turbine CCW **Drag** pushes **back** on both blades but it does not strongly affect the turbine's rotation, because that rotation is in a perpendicular plane)



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That explains **slowly** moving sailboats and **slowly** rotating Danish turbines

But when boats or blades speed up it gets more complicated: Remember: Tip Speed Ratio (TSR) = Tip (or boat) Speed / Wind Speed As shown above, TSR for modern crosswind sailing sailboats can be ~ 1.5 For modern "Danish" turbines TSR is even larger, typically ~ 5 Sailboat and turbine tip speeds thus often **exceed** wind speed! We then need to deal with "Apparent wind" Apparent wind = Wind as perceived by the sailboat or turbine blade When boats or blades speed up, **Apparent wind shifts away from True wind:** Wind speed = 0: For sailboat or blade speed equal to Wind speed: Apparent True True Wind Wind Blade (or boat) velocity Wind - Blade (or boat) velocity Apparent Wind

It is the apparent wind that determines the lift and drag on an object Which means that we really need to take a closer look at lift and drag, including: How they change with the **direction** of the apparent wind How they change with **magnitude** of the apparent wind

And we need to re-examine my earlier diagram of airfoil lift and drag



Which alleged that:

Drag just increased with airfoil tilt, all the way to 90°

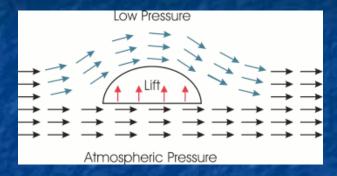
But lift first increased, but then declined

In re-examining lift I encountered something that may surprise you

(it certainly surprised me!)

The common textbook & encyclopedia explanation of lift is WRONG!

That explanation (below) focuses on the airflow over asymmetric ("humped") airfoils:



http://large.stanford.edu/courses/ 2007/ph210/glownia2/

The airfoil divides the airflow into two parts, one above and one below the airfoil
For smooth / non-turbulent (laminar) airflow, the path above is longer
Molecules that started side-by-side, must meet back up again after the airfoil
Thus, molecules following the longer upper path must move faster than those below
Then, invoke the obscure & wildly non-intuitive Bernoulli's Equation: Air_Pressure + ½ ρ v² + ρ g h = Constant (ρ = air density, v = air speed, g earth's surface gravity, h = height)

RESULT: Because v is faster **above** the airfoil, pressure must be lower => Lift

Failings of that explanation:

As noted (long ago!) in papers written by aerodynamicists in "The Physics Teacher," one by a retired career NASA engineer / project leader 1 one by a retired aero professor from the best known U.S. aero university: ² 1) Separated molecules do NOT have to meet up at the tail of the airfoil And, in fact, flow above the airfoil is often faster than predicted by Bernoulli 2) Bernoulli's Equation is not a physical law = it's not the CAUSE of things It just reports what happens in certain special situations (i.e., it's an EFFECT) 3) It describes "dry water" = Idealized incompressible zero viscosity fluids ³ 4) As applied (above) to airfoils it actually violates Newton's 3rd Law: "Reaction" (airfoil lift) invokes no "Action" (downward force upon the airflow) 5) AND THE WHOPPER: Airplane's can fly with flat and/or symmetric wings 1) "Bernoulli and Newton in Fluid Mechanics" - Norman F. Smith, The Physics Teacher 10, pp. 451-455 (1972) 2) "An Aerodynamicist's View of Lift, Bernoulli, and Newton" – Charles N. Eastlake, The Physics Teacher 40, p. 166-173,(2002)

3) The Feynman Lectures on Physics, Volume III, Chapter 40 (1964)

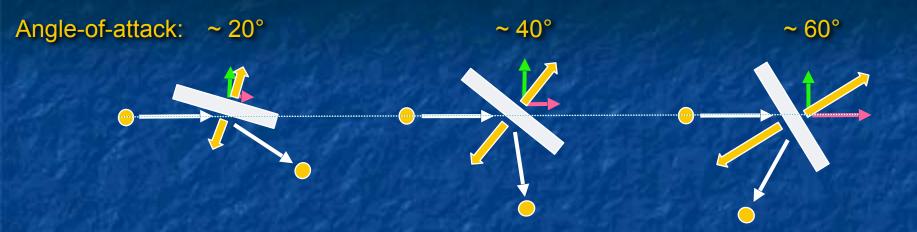
Intuition (and Newton) instead provide a simpler explanation:

Step 1:

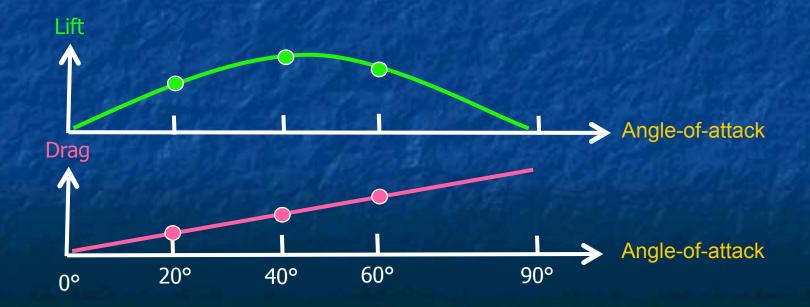
Object's "Reaction"

Consider the behavior of a bouncing ping pong ball: A ping pong ball is shot at an inclined object It is deflected downward The ball's momentum has been changed **Ball's** "Action" so the ball experienced a force (= an **action**) Newton insists that **object** had to experience a balancing force (a **reaction**) The part of that reaction **parallel** to the ball's incoming motion = **Drag** The part **perpendicular** to the ball's incoming motion = Lift

Step 2: Vary the angles-of-attack (by varying the object's tilt):

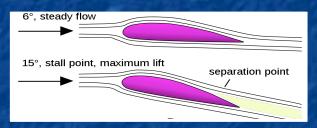


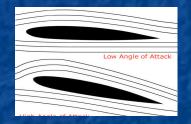
Then generate plots of how the Lift and Drag components of the reaction changed:



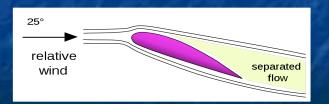
Step 3: Correct for the differences between ping pong balls and air: Like ping pong balls, air molecules DO bounce off things, transferring momentum But they also do a lot of bouncing off of one another Momentum is thus rapidly shared and redistributed among them Yielding the sort of merged / cooperative behavior typical of a gas (or liquid)

When mildly deflected, these "fluids" flow in smooth stable "streamlines"





But when strongly deflected, flow becomes unstable and confused



Turbulent airflow in greater detail:



Turbulent airflow transfers far less momentum to the object

Left: BGoogle Images link

Right: http://www.spacesafetymagazine.com/aerospace-engineering/spacecraft-design/what-is-a-stall/

Turbulent flow:

For what sort of deflections (angles-of-attack) does turbulent flow set in? Slowly moving wings induce turbulence for angles-of-attack greater than ~ 10° Rapidly moving wings induce turbulence for angles-of-attack greater than ~ 20° **How does turbulent air flow change Lift and Drag?** For Lift, the effect is a catastrophic collapse (called stall): 1-3

For **Drag**, the effect is much harder to pin down:

Because airplanes fall out of the sky once stall sets in

10 - 20°

aerodynamicists don't spend much time studying those angles-of-attack!

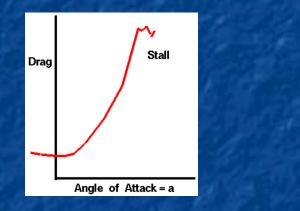
Angle-of-attack

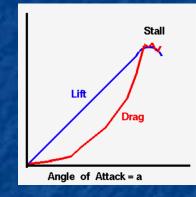
But the "wings" of ALL vertical axis wind turbines (VAWTs)

circle through ALL possible angles-of-attack

https://en.wikipedia.org/wiki/Angle_of_attack
 https://en.m.wikipedia.org/wiki/Stall_(fluid_mechanics)
 http://www.spacesafetymagazine.com/aerospace-engineering/spacecraft-design/what-is-a-stall/

I thus dug more deeply into drag under turbulent flow / stall: A lot of sources effectively ignored stall's effect upon drag, just plotting its smooth increase all the way up to 90° angle-of-attack But I ultimately discovered two NASA websites with plots suggesting that, at least immediately above stall, drag saturates: 1,2



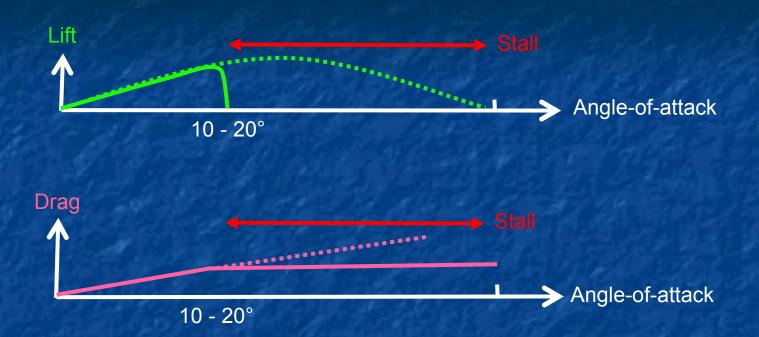


Extrapolating NASA's result, I'll thus modify my earlier **Drag** plot as follows:



Left: https://www.grc.nasa.gov/www/k-12/airplane/inclind.html RIGHT: https://www.grc.nasa.gov/WWW/K-12/airplane/kiteincl.html

Which brings us to these revised estimates of Lift and Drag:



With which we can now more fully explain the operation

and optimization of various types of wind turbine

An Introduction to Sustainable Energy Systems: WeCanFigureThisOut.org/ENERGY/Energy_home.htm

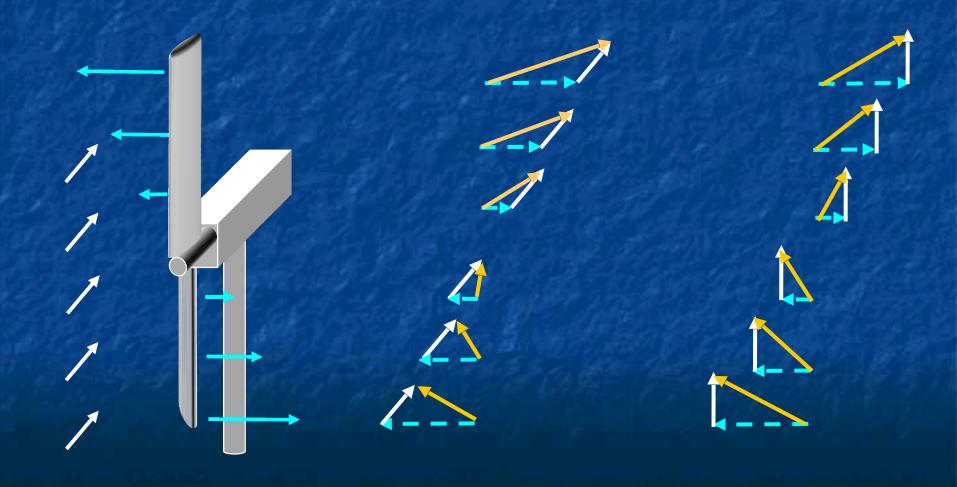
Applying these to an already-rotating Danish wind turbine:

Here, depicting it operating at a Tip Speed Ratio of ~ 2

True Wind & Blade Velocity:

Apparent Wind (viewed in perspective):

Apparent Wind (viewed from above):



Moving out toward the tips of the turbine blade:

1) The magnitude of the apparent wind increases

2) The **direction** of the **apparent wind** swings into the plane of blade rotation To capitalize upon this, **while avoiding stall**, the blades should be twisted:

> Apparent wind + blade cross-sections from above (if both blades were in fact twisted smoothly):

> > Note how this blade twist ensures that the airfoil is always tilted slightly upward from apparent wind yielding good lift (by avoiding stall angles-of-attack)

Cross-sections of a computer-optimized twisted turbine blade:

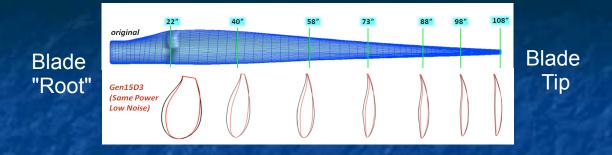


Photo of a turbine with two such blades:



Aircraft and boat propellers **twisted for same reasons**:



Top: https://crunch.craft-tech.com/applications/design-optimization/wind-turbine-blade/ Middle: https://www.windflow.co.nz/turbines/why-two-blades Bottom left: https://whirlwindpropellers.com/windtunnel/ground-adjustable-aircraft-propellers/ Bottom right: https://www.boatownersworld.com/quicksilver-silverado-vengeance-stainless-steel-boat-propeller.html

Important factoid: **Stall** can save wind turbines: Storms have torn wind turbines to pieces The problem is not so much that blade bending => breaking (induced by drag) It is instead excess winds rotating the turbine so fast that centripetal force on individual blades rips them right out of the turbine's hub But the Lift turning the turbine is driven by Apparent Wind Which is vector difference of: **True wind – Blade velocity**: As a storm drives True wind higher, it can increasingly dominate the Apparent wind, shifting Apparent wind direction, **increasing** Angle-of-attack on turbine blades which can then push those blades past their peak **Lift** and into **stall**: I ift

10 - 20°

Angle-of-attack

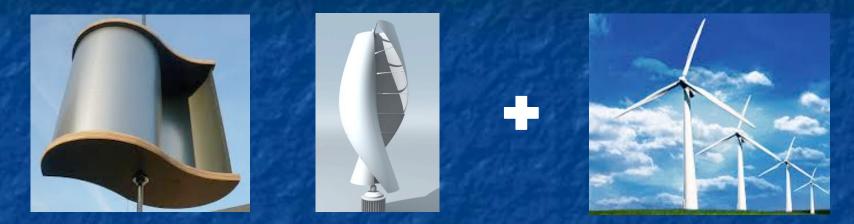
That stall provides a degree of "passive" over-speed protection "Passive" in the sense that it doesn't require sensors and alarms to set it off But to keep turbines pointing into the wind, commercial turbines already have sensors In extreme winds, they can set off stronger **active over-speed protection**: **FIRST**: Blades can be rotated about their axes (i.e., changing their **pitch**) to **zero out** wind's Angle-of-attack, slashing blade lift (called **feathering**)



SECOND: The whole head of the turbine (its nacelle) can be rotated 90° so that wind strikes it from the side, radically slowing or stopping rotation (While also minimizing total wind drag upon the complete structure)

Photo: http://eng-electric.blogspot.com/2016/11/power-control-of-wind-turbines.html

Simple **drag** thus explains **Savonius** turbines While a combination of **drag** + **lift** explains **Danish** turbines



But what about the **Darrieus** turbines that so intrigue inventors & the press?



Simple Darrieus



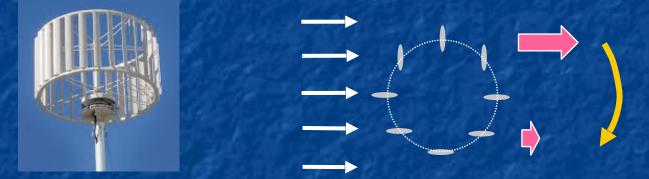
Eggbeater Darrieus



Spiral Darrieus

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Take a closer look at the Simple Darrieus turbine design From this picture, you might suspect that it twists its blades as it turns which could produce the blade cross-sections shown below right, which **DOES** create strong drag on one side, light on the other => Rotation

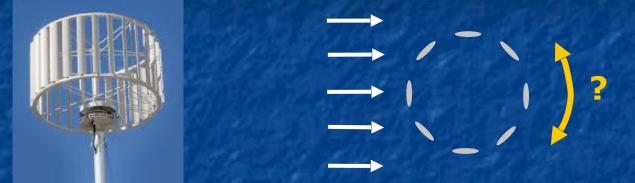


But how would you get the blades to twist?

You could add motors on each blade (likely impractical / certainly expensive) Or you might devise some clever way of linking the blades together such that they cause each other to pivot as the whole assembly rotates That HAS been tried, but based solely on drag, it is then not a true Darrieus turbine It's more like a distant cousin of the drag-based Savonius turbine

A true Darrieus turbine exploits Lift

And a Simple Darrieus turbine is **quite** simple, with just one rigid rotating assembly:



Is its rotation then induced by some subtlety in its blade cross-sections? No, simple flat plates will work (e.g., a rolled up set of wood-slat venetian blinds) But how then does wind cause a Darrieus turbine to rotate? In fact: WIND WON'T START A DARRIEUS TURBINE ROTATING But if it's ALREADY rotating, wind WILL SPEED UP THAT ROTATION The latter is something aerodynamicists only ever explain to fellow aerodynamicists But after a lot of study, I believe I can now explain it to you using diagrams:

We need to estimate lift & drag all around the full Darrieus circle:

Producing Apparent wind diagrams as input to lift & drag Angle-of-attack Diagrams



Fortunately, for a Simple Darrieus turbine we need only consider one horizontal plane Because its winds flow only within horizontal planes, producing forces entirely within those same horizontal planes, (making all horizontal planes effectively equivalent) Further, analysis of wind striking a non-rotating Simple Darrieus turbine is "trivial" because symmetry tells us that everything happening on one side of the turbine is balanced (and thus canceled) by what's happening on the other side:

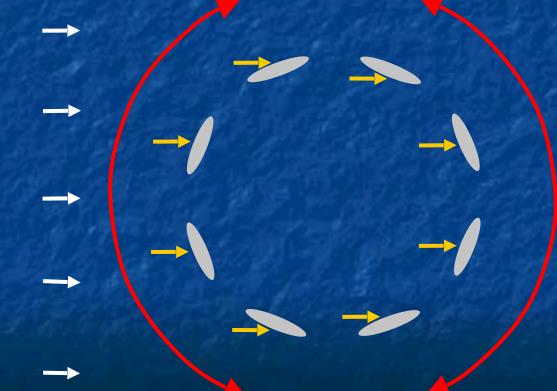
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NON-ROTATING Darrieus turbine: Winds and where blades stall

With no rotation: Apparent Wind = Real Wind

But then, for most of the blades, Angles-of-attack are >> 20°,

inducing massive turbulence, putting those blades in full aerodynamic stall:



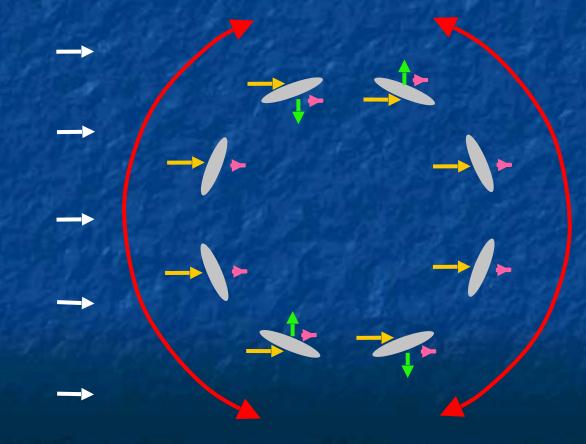
Passing thru these sectors, blades stall

NON-ROTATING Darrieus turbine: Blade lift & stall forces

Using those apparent wind angles-of-attack as input

to the lift and drag diagrams two slides above (which accounted for stall),

the lift and drag vectors for individual blade can be estimated as:

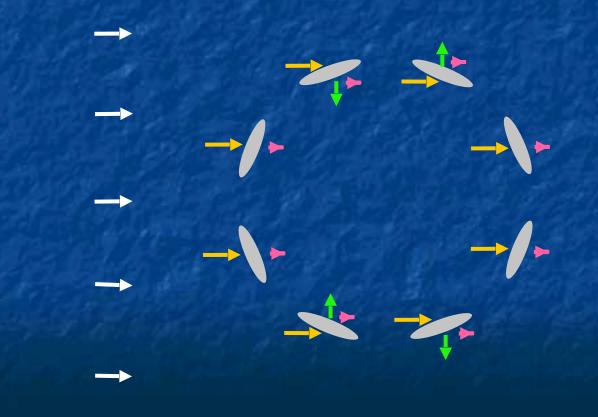


NON-ROTATING Darrieus turbine: Cumulative effect of those forces

There is COMPLETE top to bottom symmetry:

EVERY force in the top half is precisely mirrored in the bottom half

Torques thus precisely cancel => The turbine will NOT start rotating



Repeating that analysis for an ALREADY-ROTATING Darrieus turbine:
Which is much harder because now not only does Apparent Wind ≠ Real Wind, but Apparent Wind changes both direction AND magnitude around the circle
Let's say the turbine is now rotating with Blade Speed = True Wind Speed (TSR =1) Meaning they have the same magnitude, but not necessarily the same direction
True Wind = → Blade velocity = → ↓ ↓ ↓ ↓ ↓ ↓ ↓

Then, moving around the circle, **True Wind** – **Blade velocity** => **Apparent Wind**

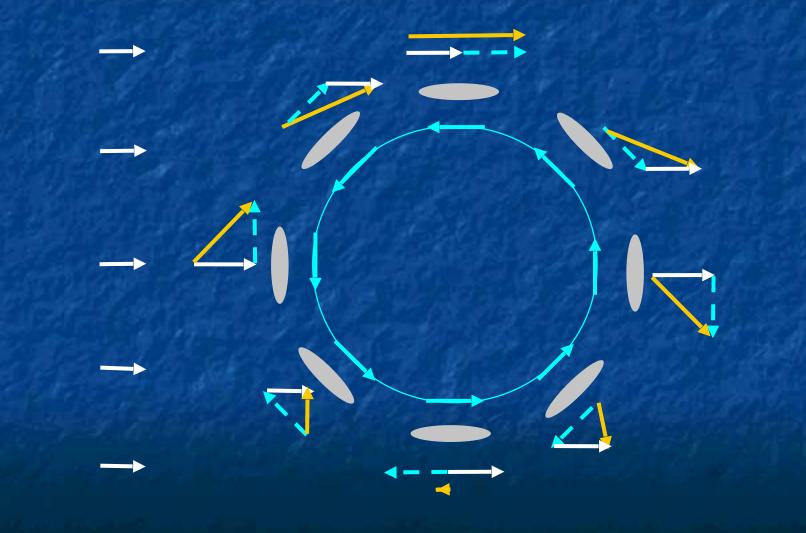
At one point on that circle things will look like this:



Diving right in:

ALREADY-ROTATING Darrieus turbine: Winds

Key: True Wind Rotation (solid) Rotational Wind (dashed) Apparent Wind



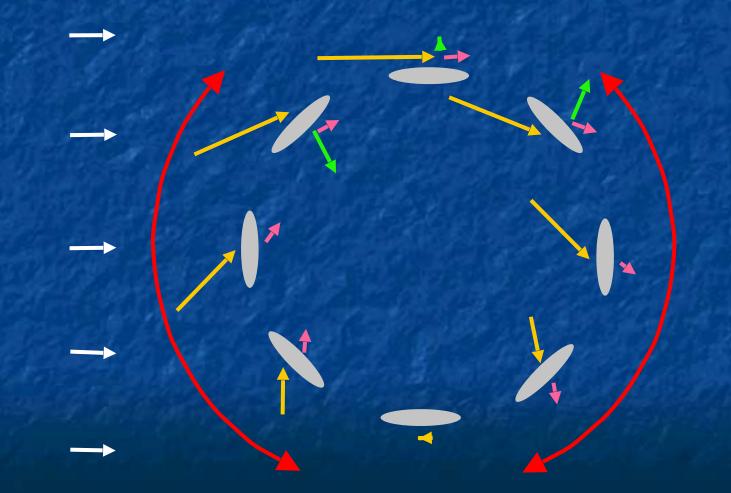
ALREADY-ROTATING Darrieus turbine: Where the blades stall

Note that symmetry has been been lost: Top half no longer mirrors bottom half!

Passing thru these sectors, blades stall

ALREADY-ROTATING Darrieus turbine: Blade lift & drag forces

Based on the turbulence / stall-corrected lift and drag diagrams:



ALREADY-ROTATING Darrieus turbine: Cumulative effect of those forces

Strongest forces occur when blades pass thru non-stalled top of diagram Net torque (rotational force) = CCW = Direction of turbine's initial rotation!



Wind thus causes an ALREADY-ROTATING Darrieus turbine to SPEED UP

Are you still skeptical? (I probably would be) Those plots were awfully complex and subtle (and very easy to get wrong) I thus dug up aerodynamically-rigorous computer simulations of Darrieus turbines, which calculated **precise** lift & drag for their rotating blades,

which were then used to plot torque as a blade rotates around the circle:

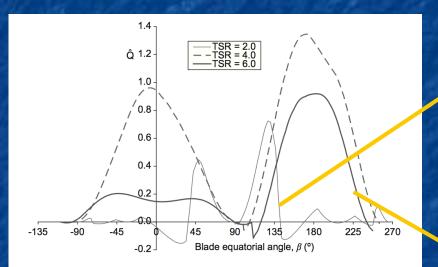


Figure 13: Non-dimensional torque coefficient, \hat{Q} , as a function of blade azimuth angle, β estimated from a double-multiple-streamtube blade element analysis of a VAWT (parameters as for Fig. 12).

For slow rotation (TSR = 2): Plot is almost left/right symmetric => Weak net torque

For fast rotation (TSR = 6): Plot is very left/right non-symmetric => Strong net torque

Figure: Development and Analysis of Vertical-axis Wind Turbines, Chapter 8, Paul Cooper, School of Mechanical, Materials and Mechatronic Engineering, University of Wollongong, NSW, Australia (<u>link</u>)

SEE ALSO: Other VAWT aerodynamic theory papers given on this note set's Resources Webpage

I REALLY wanted to figure out why Darrieus turbines spin Because it would then explain something that's mystified me for decades: How the heck / Why the heck Autogyros fly! An Autogyro is a go-cart with the not-so-secret ambition of becoming a helicopter They are often homebuilt, consisting of crude three-wheel open chasses, on which a seat and motorized pusher propeller are mounted But then, above that seat, on a wobbly / tiltable mount, a ridiculously UNPOWERED larger propeller is added:



"Little Nellie" in the 1967 James Bond film: You only Live Twice

Photo: https://en.wikipedia.org/wiki/ Wallis_WA-116_Agile When you power up an Autogyro's (Gyrocopter's) pusher prop:
You can drive around on the ground like a normal go cart,
with the floppy upper prop doing nothing other than endangering passersby
But if you FIRST stand up in your seat, and push the upper prop into slow rotation,
something miraculous occurs: (although it occurs more slowly here):





YouTube link

YouTube link

(Cached copies of these videos are also linked from this note set's Resources Webpage)

But is the rotor just turning due to airflow from below & ahead?
Which would inhibit that airflow (like a Danish wind turbine) acting a bit like a wing, thereby providing lift if driven forward fast enough by the pusher prop
The videos show that both rotors were hand-started CCW (per the added arrows)
And looking very carefully at these photos from those videos you can see that each rotor blade is inclined as in the diagram shown at the right:



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Tilts + rotational direction are NOT consistent with airflow from below:

CW Rotation responding to air flow entering from below:

Which only **retards** that upward airflow

CCW Rotation induced by a Darrieus-like airflow mechanism:

Which yields helicopter-like downward thrust

<u>†</u> † † † †



I suspect the rotor spun up so slowly in 2nd video BECUASE the pilot mistakenly tilted it such that: The (left) retardation-producing effect (=> CW rotation) was strengthened to the point that it **almost** overwhelmed the (right) thrust-producing effect (=> CCW rotation)

What then (finally) is the **best** type of wind turbine? That battle seems to shape up as: Monster Danish turbines (the overwhelming wind industry favorite) Versus

An assortment of vertical axis wind turbine (the inventor, press & public favorite)

With that inventor/press/public preference based on VAWTs' diminutive size + eye-pleasing appearance











VAWTs **might** work if more blades and/or wider blades made up for size After all, the Danish turbine's three narrow blades seem to **miss most of the wind!** But as I discussed far above:

Turbine blades have a "bow wave" making them act much larger than they are Turbine rotation then makes it ~ impossible for air to pass without interaction A turbine's effective area is thus closer to a wind's eye view of the turbine's total swept area than to the sum of individual blade areas:



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Because today, Danish turbines are SO MUCH BIGGER than almost all VAWTs

Which puts today's VAWTs at a huge disadvantage



Which means that even if whole fields or whole building rooftops were covered by VAWTs, at their current diminutive but eye-pleasing sizes, they would together intercept **so much less wind**, and, nearer to the ground, **so much slower wind**

Which means that fields or rooftops packed with small / eye-pleasing VAWTs would have to be VASTLY MORE EFFICIENT at harvesting the much smaller amount of wind that they DO intercept Which IS INDEED suggested in many (if not most) press articles about VAWTs! But ARE today's VAWTs vastly more efficient than Danish turbines? NO! But might future improved VAWT designs change the answer to YES? Our best clues from more advanced aerodynamic theory Which I will cover in this note set's **next** section: **Aerodynamics 201** But even if VAWTs fail to achieve vastly higher efficiencies than Danish turbines, there would still be the possibility of making VAWTs **as large as** Danish turbines



But Danish turbines can be \sim 150 meters in diameter, 100 meters above the ground Imagining such a huge and tall VAWT, the incredible challenges seem obvious!

But could a **medium** sized but many-bladed VAWT still win out? More blades inevitably require more material and produce more rotating mass More rotating mass then requires a stronger and heavier supporting structure All of which increase the cost of building and/or maintaining such a turbine And there is a strong **AERODYNAMIC ARGUMENT** against too many blades: More blades boost the overall drag trying to slow a turbine's rotation Which leads aerodynamicists to actually suggest the use of: 1-3 One blade (alone!) – in combination with a counterweight: OR

A large number of very narrow - and thus low drag - blades

(which, unfortunately, would be weak & susceptible to breakage)

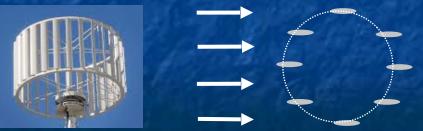
Photo: https://www.videoblocks.com/video/static-shot-of-a-single-blade-wind-turbine-on-a-windy-day-soh1f9wjliujtg2bn
 1) http://drømstørre.dk/wp-content/wind/miller/windpower%20web/en/tour/design/concepts.htm
 2) https://www.windpowermonthly.com/article/1083653/three-blades-really-better-two
 3) http://explorecuriocite.org/Explorer/ArticleId/193/why-dont-wind-turbines-have-more-than-3-blades-193.aspx

For Danish turbines the optimum is actually thought to be three blades: 1 Because the wind speed increases rapidly with height, a blade is pushed back most strongly at the top of its rotation But for two blades, as one blade tops its rotation the other is at the bottom, passing in front of the pylon Not only is wind slower at that lower height but it is particularly slow in front of the pylon's obstruction Maximum push back at top + Minimum push back at bottom => An abrupt imbalance that tries to jerk a blade pair out of vertical Which can then generate a pulse of **noise** and requires stronger/bulkier mechanical design Such top-to-bottom synchronization is avoided with 3 blades: 1) Danish Wind Industry Association:

http://drømstørre.dk/wp-content/wind/miller/windpower%20web/en/tour/design/concepts.htm

But VAWTs seem to have two **BIG** strengths . . . and one **BIG** weakness: The strengths come from the vertical rotation axis of VAWTs: Complex and heavy electrical generators can be housed on the ground Where they can can be accessed and maintained very easily Vs. climbing up to a Danish turbine's nacelle ~ 100 meters above the ground Turbines rotate in a full compass circle thus having no front (or back) So they can accept, even minute by minute, wind coming from ANY direction Vs. the Danish turbine's need for sensor and motor-driven nacelle rotation The **apparent** weakness is that wind can't get at least a Darrieus VAWT started But a little Grid power sent to its generator, then acting as a motor, does the job The **real** weakness comes from their 2nd strength: VAWTs can't turn out of the wind How then do you keep a storm from destroying a VAWT?

Danish turbines have two ways of protecting themselves from excessive wind: The nacelle (and thus the turbine) can rotate perpendicular to the wind The turbine then has minimal cross section and thus minimal wind interaction Vs. a **VAWT** that - via its basic design - is always full face to the wind The Danish turbine's other (actually its first) defense is to feather its blades Rotating blades to align their airfoils ~ parallel to the wind: Similar feathering is possible for **Simple Darrieus Turbines** It can be done by mounting each blade on off-center pivots and then adding torsion springs sized to allow each blade to pivot in high wind



This is **exactly** how blades were feathered on some early Danish turbines!

But wind-driven feathering is not possible in other Darrieus variants Further, where steady blade rotation of a Danish turbine tries to **stretch** its blades, The same rotation of a Darrieus turbine tries to **bow** its blades outward But that rotation has a cyclic variation of angle-of-attack and thus of lift & drag, so bowing due to aerodynamic forces is also cyclic, with that flexing setting up resonances that can weaken or even destroy Darrieus turbines Large Darrieus turbines are thus known for short operational lifetimes This, the 96 meter tall "Eole Cap Chat" Quebec CA turbine managed to stay in operation for only six short years It survives to this day only because (now locked in place) locals continue to maintain it as a **tourist attraction**

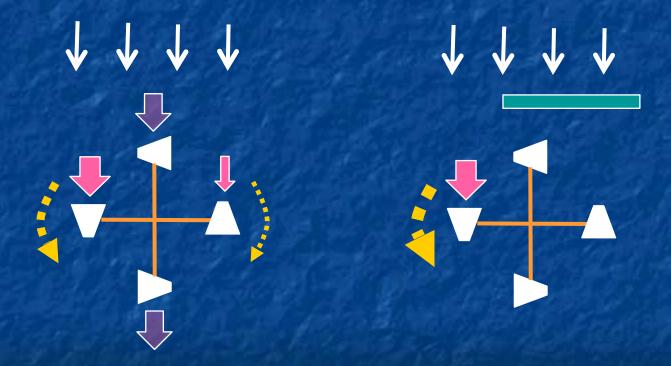
Eole Cap Chat photo: http://www.odec.ca/projects/2011/likek2/windturbines.html

How might future VAWTs be improved?

Starting with Savonius turbines:

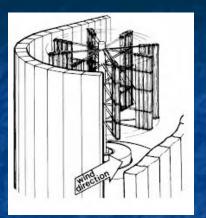
Recall that they spin based solely on a side-to-side imbalance of drag forces

They'd work better if that imbalance were increased by blocking half their wind:



Archaeologists have found evidence that this HAS been tried:

Artist's depiction of Persian "Panemone" Savonius turbine – Pre 1000 AD: 1



One assumes this ancient turbine was used only for pumping water or grinding grain

Figure: "Wind Technology: Historical Drive Trains, Conversion Devices, Configurations, Design" http://www.wind-works.org/cms/fileadmin/user_upload/Files/presentations/Wind-101/Wind_101half-6_Technology_01.pdf

1) https://en.wikipedia.org/wiki/Panemone windmill

However, using only drag, Savonius turbines can **at best** reach wind speed (TSR \leq 1)

Making them the "windjammers" of wind power:



But higher blade speed => Higher turbine kinetic energy => HIGHER POWER OUT

Purely drag turbines are thus doomed to very low conversion efficiencies

Savonius turbines achieve conversion efficiencies of 7, 9, 11% ²⁻⁴

2) https://aip.scitation.org/doi/pdf/10.1063/1.5024100 3) https://ac.els-cdn.com/S111001681200049X/1-s2.0-S111001681200049X-main.pdf?_tid=e63cb153-20c9-4c08-871de58db583d95e&acdnat=1535037457_2f9e50aa620f3e80abedede5fa1f1fb6 4) http://www.engr.mun.ca/~blaines/Docs/Final%20Report-April-09.pdf Danish and Darrieus turbines instead exploit both drag and lift:

Making them more like modern Hobie Cat sailboats:



Danish turbines typically operate at TSR ~ 5, and can reach TSR ~ 10
While aerodynamicists model Darrieus turbines achieving TSR = 2-8 1
What power conversion efficiencies (power coefficients, C_p) do they achieve?
Danish turbines are usually cited as achieving ~ 50% conversion efficiency
Darrieus turbines are usually cited as "now having lower efficiency"
From sources spanning real-life data, wind tunnel tests & computer simulations, but avoiding sources with obvious biases or vested interests, I found:

Danish turbines achieve efficiencies of 45-50, 48, 50% ²⁻⁴ Darrieus turbines achieve efficiencies of 20, 35-40, 40% ⁵⁻⁷

1) As shown above at the end of my explanation of how Darrieus turbines work (~16 slides back) 2) https://en.wikipedia.org/wiki/Betz%27s_law 3) https://www.researchgate.net/publication/289639250/download 4) http://css.umich.edu/factsheets/wind-energy-factsheet 5) http://iopscience.iop.org/article/10.1088/1742-6596/923/1/012036/pdf 6) http://iopscience.iop.org/article/10.1088/1742-6596/753/6/062009/pdf 7) https://prod.sandia.gov/techlib-noauth/access-control.cgi/1980/800179.pdf

Those numbers already torpedo the "fields of tiny VAWTs" scenario: Based on conservation of energy, power conversion efficiency can only rise to 100% With Danish turbines already achieving ~ 50% efficiency No future VAWT could possibly do better than doubling that efficiency Which means that even with almost magically efficient future VAWTs their effective sizes would have to total at least half that of a Danish turbine in order to compete with that Danish turbine's output power

Thus, instead of this choice:

The choice would be:



What are the chances of **any** turbine approaching 100% efficiency? VAWTs already exist in so many different variations that predicting the cleverness of future inventors is clearly impossible We must therefore look to nature to define an upper limit, which here means Newton's laws as manifested in advanced aerodynamics To explain that natural limit, I will now need to: Explain how **Bernoulli's Equation** is derived, and then use **it** to derive: **Betz's Law** providing a theoretical limit on lift + drag turbine efficiency Then derive the companion theoretical limit on pure drag turbine efficiency Yes, it is about to get quite technical, but even if you fear it will get too technical, I urge you to scan the next section's text explanations & figures And at least review its two conclusion slides before reading on to: MY SUBSEQUENT SECTION ABOUT WHICH TURBINES SURVIVE IN THE REAL WORLD

AERODYNAMICS 201:

Bernoulli's Equation describing air flows The Betz theoretical limit on lift/drag turbines The corresponding theoretical limit on pure drag turbines

Bernoulli's Equation relating pressure & velocity:

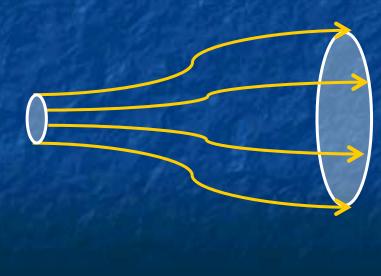
Bernoulli's equation models the flow of "fluids" including both liquids & gases Those flows can be exceedingly complex phenomena Which led Bernoulli to employ a pair of approximations, and to restrict his analysis to a special situation The 1st approximation is that the fluid's viscosity is ignored That is a reasonable approximation for low pressure gases, a so-so approximation for medium pressure gases (such as our air), and a lousy approximation for at least rapidly moving liquids The restriction is that the fluid must be is in steady, non-turbulent flow Its molecules then move along constant smooth paths, called **streamlines** Which is equivalent to saying that their flow is laminar

Zero viscosity laminar flow is depicted in this diagram: The yellow lines are some of the streamlines along which the molecules flow We will track molecules entering through the left oval area And exiting through the right oval area And then calculate how things change between those two places With the "things" tracked including these (given in both words and symbols):

Flow In:

Pressure = Pressure_{in} Mass per volume = ρ_{in}

Area = A_{in} Velocity = v_{in} Mass Flow = MF_{in} Power = P_{in}



Flow Out:

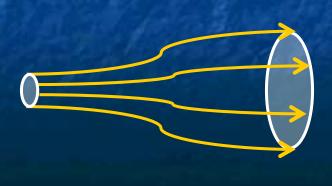
Pressure = Pressure_{out} Mass per volume = ρ_{out}

Area = A_{out} Velocity = v_{out} Mass Flow = MF_{out} Power = P_{out}

Properties of that flow – here assumed to be air (keeping diagram handy): ENERGY PER AIR MASS = Kinetic Energy + Potential Energy Kinetic energy per gas mass = $\frac{1}{2}$ v² Gravitational potential energy per mass = g x height thus: Energy per air mass = $\frac{1}{2}v^2 + gh$ (1)MASS FLOW and CONSERVATION OF MASS In steady flow, air mass neither accumulates or diminishes at any point MASS FLOW = (mass/volume) (area) (velocity) = $\rho A v$ = Constant Thus: Mass Flow_{in} = Mass Flow_{out} OR: $\rho_{in} A_{in} v_{in} = \rho_{out} A_{out} v_{out}$ (2)

> Pressure = $Pres_n$ Mass per volume = ρ_{in}

Area = A_{in} Velocity = v_{in} Mass Flow = MF_{in} Power = Power_{in}



Pressure = $Pres_{out}$ Mass per volume = ρ_{out}

Area = A_{out} Velocity = v_{out} Mass Flow = MF_{out} Power = Power_{out} That air flow can do work (e.g., by powering a wind turbine)

That work can be calculated from the change in the AIR FLOW's POWER Air Flow Power = Pressure x Area x Air velocity thru that area: <u>Thus: Air Flow Power_{in} = Pressure_{in} A_{in} v_{in}</u>

And: Air Flow Power_{out} = Pressure_{out} A_{out} V_{out}

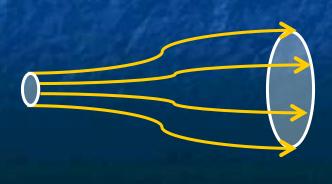
The work done by that air flow is then:

 Δ Power = Pressure_{in} A_{in} v_{in} - Pressure_{out} A_{out} v_{out}

(3)

Pressure = $Pres_n$ Mass per volume = ρ_{in}

Area = A_{in} Velocity = v_{in} Mass Flow = MF_{in} Power = Power_{in}



Pressure = $Pres_{out}$ Mass per volume = ρ_{out}

Area = A_{out} Velocity = v_{out} Mass Flow = MF_{out} Power = Power_{out}

But that power had to come out of the air molecules' energy Net molecular power flow = (Mass Flow) x (Energy per air mass) Inserting mass flow (2) and energy density (1) from two pages back $Power_{in} = \rho_{in} A_{in} v_{in} (1/2 v_{in}^2 + g h_{in})$ $Power_{out} = \rho_{in} A_{in} v_{in} (1/2 v_{out}^2 + g h_{out})$ The difference between power flow out and power flow in is then: $\Delta \text{Power} = \rho_{\text{in}} A_{\text{in}} v_{\text{in}} (1/2 v_{\text{out}}^2 + g h_{\text{out}}) - \rho_{\text{in}} A_{\text{in}} v_{\text{in}} (1/2 v_{\text{in}}^2 + g h_{\text{in}}) (4)$ Setting that equal to work done by that air flow (equation 3): $\operatorname{Pres}_{\operatorname{in}} A_{\operatorname{iin}} v_{\operatorname{in}} - \operatorname{Pres}_{\operatorname{out}} A_{\operatorname{out}} v_{\operatorname{out}} = \rho_{\operatorname{in}} A_{\operatorname{in}} v_{\operatorname{in}} (1/2 v_{\operatorname{out}}^2 + g h_{\operatorname{out}}) - \rho_{\operatorname{in}} A_{\operatorname{in}} v_{\operatorname{in}} (1/2 v_{\operatorname{in}}^2 + g h_{\operatorname{in}})$ Dividing through by the constant Mass Flow = $\rho_{in} A_{in} v_{in} = \rho_{in} A_{in} v_{in}$ $(\text{Pressure}_{\text{in}} / \rho_{\text{in}}) - (\text{Pressure}_{\text{out}} / \rho_{\text{out}}) = (1/2 \text{ v}_{\text{out}}^2 + g \text{ h}_{\text{out}}) - (1/2 \text{ v}_{\text{in}}^2 + g \text{ h}_{\text{in}})$

That equation is then further simplified by making a 2nd approximation: It is assumed that the density of the "fluid" never changes significantly That's a very good assumption for liquids such as "incompressible" water: Even though its pressure increases one atmosphere per 10 meters of depth, deep, deep water is only slightly denser than near-surface water But high school's "Ideal Gas Law" (PV = nRT) taught us that gases CAN compress However: Air flowing slowly by an obstruction may not compress ... much Because it then has time to instead detour around that obstruction But how "slow" is slow? Sonic booms occur when air **can no longer** flow out of a jet's way, And air has no choice but to pile up (compress) in front of that jet A Rule of Thumb: Air compression is minor at velocities << Speed of sound

Negligible viscosity + Negligible compression = "Dry Water"

Which, as weird and implausible as it sounds, IS the basis for simple aerodynamics For such "dry water" the fluid density is ~ constant: $\rho_{in} = \rho_{out} = \rho$ Then, if the change in flow height is small ($h_{in} \sim h_{out} = h$), equation (5) becomes: Pressure_{out} / ρ + $\frac{1}{2}$ v_{out}² = Pressure_{in} / ρ + $\frac{1}{2}$ v_{in}² But the "in" and "out" areas (the ovals) were arbitrarily positioned, and could be located anywhere along the air flow streamlines IMPLYING: **Anywhere** along a streamline, (Pressure_{in} / $\rho + \frac{1}{2} v_{in}^2$) must be the same:

For a non-viscous incompressible fluid in laminar flow (along streamlines):

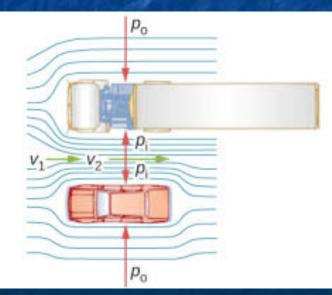
Pressure / density + ¹/₂ velocity² = A constant Bernoulli's Equation

For a low-viscosity almost incompressible fluid, this should be approximately correct

One of the Bernoulli Equation's strange predictions:

Driving along freeway, have you ever been surprised by a big truck overtaking you? Appearing suddenly out of your blind spot, it's lurking size can startle you But have you then imagined that the truck was sucking you toward it? Bernoulli's Equation says that it was not your imagination:

Passing your vehicles, the airflow had to speed up, particularly between them



With the highest speed **between** the vehicles Bernoulli's Equation: **Pressure / p + ½ velocity² = constant** Says that there the air pressure IS lowest (A passing truck does indeed suck!)

Figure: https://phys.libretexts.org/TextBooks_and_TextMaps/University_Physics/Book%3A_University_Physics_(OpenStax)/ Map%3A_University_Physics_I_-_Mechanics%2C_Sound%2C_Oscillations%2C_and_Waves_(OpenStax)/ 14%3A_Fluid_Mechanics/14.6%3A_Bernoulli%E2%80%99s_Equation

The Betz Limit on wind turbine efficiency (C_p):

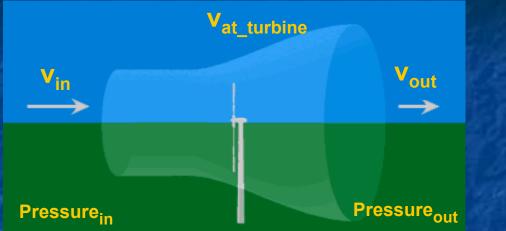


Figure: My rework of WindPower.Org's: http:// ele.aut.ac.ir/~wind/en/stat/ betzpro.htm

This diagram illustrates air flow passing by a wind turbine As we now know, air compresses little when travelling at subsonic speeds But given that mass flow = (density = ρ) (Area of cross-section) (velocity): With ρ ~ constant, as the air's velocity decreases passing the turbine, flow cross section must increase to maintain mass flow across the diagram Only then can: $\rho A_{in} v_{in} = \rho A_{turbine} v_{at_turbine} = \rho A_{out} v_{out}$

Calculations would now be easy if we knew values AT the turbine Newton pretty much supplies the answers: The force on the turbine equals its area x the air pressure difference front to back: Force = $A_{turbine} \Delta Pressure \sim A_{turbine} (Pressure_{in} - Pressure_{out})$ (1)That force must also equal the rate at which the air is losing momentum, which equals its mass flow x its change in velocity: = dMomentum / dt = (Mass Flow) $(v_{out} - v_{in})$ Force = ρ (Air Flow) ($v_{in} - v_{out}$) = $\rho A_{turbine} v_{at turbine} (v_{in} - v_{out})$ (2)The power transferred from the air flow into the turbine would then be that force multiplied by how fast air arrives at the turbine: Power_{turbine} = Force x v_{at turbine} (3)

We now use Bernoulli's Equation to pin down velocity AT the turbine: Bernoulli's Equation says along streamlines (Pressure / $\rho + \frac{1}{2} v^{2}$) is constant, thus: Pressure_{in} + $\rho v_{in}^2 / 2$ = Pressure_{out} + $\rho v_{out}^2 / 2$ **Reworking**: Pressure_{in} - Pressure_{out} = $\rho v_{out}^2 / 2 - \rho v_{in}^2 / 2$ Substituting that into the **first force equation (1)**: $A_{at \ turbine} \left(\rho v_{out}^{2} / 2 - \rho v_{in}^{2} / 2\right) = \left(A_{turbine} v_{at \ turbine} \rho\right) \left(v_{out} - v_{in}\right)$ After some cancellations and rearrangement, that becomes: But $D^2 - E^2 = (D-E)(D+E)$, Yielding: $v_{out}^2 - v_{in}^2 = 2 v_{turbine} (v_{out} - v_{in})$ $v_{at turbine} = (v_{out} + v_{in}) / 2$ (4)

You would have guessed that v_{at_turbine} must be **between** v_{out} and v_{in} But without Bernoulli there is no reason to assume it is their **exact** average (As **IS** slyly assumed on many, many wind power information websites!)

Putting it all together to get a turbine power limit:

Solving the preceding $v_{at_turbine}$ equation for v_{out} : $v_{out} = 2 v_{at_turbine} - v_{in}$

Using this to eliminate v_{out} from **second force equation (2)**

Force = $\rho A_{turbine} v_{at_turbine} (v_{in} - v_{out}) => 2 \rho A_{turbine} v_{at_turbine} (v_{in} - v_{at_turbine})$

Substituting that force into the power equation (3)

Power_{turbine} = Force x $v_{at_turbine}$ = 2 $\rho A_{turbine} v_{at_turbine}^2 (v_{in} - v_{at_turbine})$

= 2
$$\rho A_{turbine} (v_{turbine}^2 v_{in} - v_{turbine}^3)$$

Solving for the power maximum with respect to possible values of v_{at turbine}:

d Power_{turbine} / d v_{at_trubine} = 0 = 2 $\rho A_{turbine}$ (2 v_{at_turbine} v_{in} - 3 v_{at_turbine}²)

The maximum thus occurs when $v_{at turbine} = (2/3) v_{in}$

Substituting that optimum $v_{at turbine}$ back into the power equation (5)

Power_{turbine max} = $2\rho A_{turbine}((4/9)v_{in}^3 - (8/27)v_{in}^3) = (8/27) \rho A_{turbine}v_{in}^3$

(5)

Comparing that to the wind's **total** power:

As derived at the opening of this note set: Chunk of "air flow" Energy Density $\frac{1}{2} \rho v^2$ Turbine generator (with whirly things)

The kinetic energy per volume of wind = $\frac{1}{2}$ (M / volume) $v^2 = \frac{1}{2} \rho v^2$

But wind volumes arrive at the turbine at a rate proportional to the wind's velocity

So the wind's total power = $\frac{1}{2} \rho A_{turbine} V_{in}^{3}$

Dividing that into the maximum turbine power that we just calculated:

Max Turbine Efficiency (C_{P_max}) = [(8/27) $\rho A_{turbine} v_{in}^3$] / [½ $\rho A_{turbine} v_{in}^3$] = 16 / 27 => 59.3% = THE BETZ LIMIT

Max wind kinetic energy to turbine blade kinetic energy conversion efficiency

Limit on purely drag wind turbine efficiency (C_p):

The derivation of this (un-named) limit resembles the Betz derivation:

According to Newton, Force = Rate of momentum transfer

The momentum per volume of an air flow = ρv_{air}

The volume of flow passing through a surface per time = $A_{surface} v_{air}$

If the air's full momentum were transferred to that surface, force on it would be

Force = $\rho A_{surface} v_{air}^2$

In real situations only some fraction of that momentum is transferred, thus Force = (C_D / 2) $\rho A_{surface} v_{air}^2$

This equation defines what is called **the Drag Coefficient**, C_D Why stick in an additional 2? I suspect so that many C_D 's end up being ~ 1

Values of the drag coefficient for common shapes:

Now imagine one such object (a turbine blade) being dragged by an air flow The air flow's velocity is: vair Drag Shape Coefficient Sphere \longrightarrow 0.47 It drags the object along at a velocity of: v_{object} Half-sphere \longrightarrow (0.42 Cone \rightarrow The object experiences an Apparent wind of (v_{air} - v_{object}) 0.50 1.05 Making the drag force on the object: $\begin{array}{c} \text{Angled} \\ \text{Cube} \end{array} \longrightarrow \bigotimes$ 0.80 Force = (C_D / 2) $\rho A_{surface} (v_{air} - v_{object})^2$ Long Cylinder 0.82 $\stackrel{\text{Short}}{\longrightarrow} \square$ 1.15 Referring back to Newton, the work done by a force is: $\frac{\text{Streamlined}}{\text{Body}} \longrightarrow \bigcirc 0.04$ Work = Force x (distance object is moved by that force) Streamlined Half-body 0.09 = Energy transferred to object Measured Drag Coefficients https://en.wikipedia.org/wiki/ Drag coefficient Power to the object is the time derivative of energy, thus: **Power** = Force x d/dt (distance object travels) = Force x v_{object} = $(C_D / 2) \rho A_{surface} (v_{air} - v_{object})^2 v_{object}$ (1)

For what object velocity will power transfer be maximized? It will be a maximum when: $d / d v_{object}$ (Power) = 0 Thus: = d / d v_{object} (C_D / 2) ρA_{surface} ($v_{\text{air}} - v_{\text{object}}$)² v_{object} 0 = (C_D / 2) ρ A_{surface} [- 2 (v_{air} - v_{object}) v_{object} + (v_{air} - v_{object})²] = (C_D / 2) ρ A_{surface} [3 v_{object}² - 4 V_{air} V_{object} + v_{air}²] Requiring: $3 v_{object}^2 - 4 V_{air} V_{object} + v_{air}^2 = 0$ With solution: $v_{object} = v_{air} / 3$ Inserting this optimum object speed back into the power equation (1): $Power_{max} = (C_D / 2) \rho A_{surface} (v_{air} - v_{air} / 3)^2 (v_{air} / 3)$ = $(4/27 \text{ C}_{\text{D}}) \times (1/2) \rho A_{\text{surface}} v_{\text{air}}^{3} = (4/27 \text{ C}_{\text{D}}) \times \text{Air's Power}$ Thus: Max drag turbine efficiency ($C_{P max}$) = (4/27) C_{D} For typical objects with drag coefficients near 1, $C_{P max}$ is ~4 / 27 = 14.8 %

Conclusions from Aerodynamics 201:

Maximum theoretical power coefficients (transfer of kinetic energy, air to turbine): Betz Limit for lift + drag turbines = 16 / 27 = 59.3 % Limit for pure drag turbines ~ 4 / 27 = 14.8 %

Versus real-life and/or computer simulated power coefficients that I reported earlier: Danish (lift + drag) turbines: <u>44-50, 48, 50%</u> ¹⁻³

Darrieus (lift + drag) turbines:

20, 35-40, 40% 4-6

Savonius (pure drag) turbines:

7, 9, 11% ⁷⁻⁹

1) https://en.wikipedia.org/wiki/Betz%27s_law

- 2) https://www.researchgate.net/publication/289639250/download
- 3) http://css.umich.edu/factsheets/wind-energy-factsheet

4) http://iopscience.iop.org/article/10.1088/1742-6596/923/1/012036/pdf
5) http://iopscience.iop.org/article/10.1088/1742-6596/753/6/062009/pdf
6) https://prod.sandia.gov/techlib-noauth/access-control.cgi/1980/800179.pdf

7) https://aip.scitation.org/doi/pdf/10.1063/1.5024100

8) https://ac.els-cdn.com/S111001681200049X/1-s2.0-S111001681200049X-main.pdf?_tid=e63cb153-20c9-4c08-871de58db583d95e&acdnat=1535037457_2f9e50aa620f3e80abedede5f a1f1fb6

9) http://www.engr.mun.ca/~blaines/Docs/Final%20Report-April-09.pdf

Conclusions from Aerodynamics 201 (cont'd):

Today's Danish turbines are at ~ 5/6 th of their theoretical limit Today's Darrieus turbines are at ~ 2/3 ^{rds} of their theoretical limit But even if they reached that full limit (while Danish turbines were stagnant) To produce power equal to a Danish turbine, their size would have to be ~ 5/6 th that of a comparably powerful Danish turbine

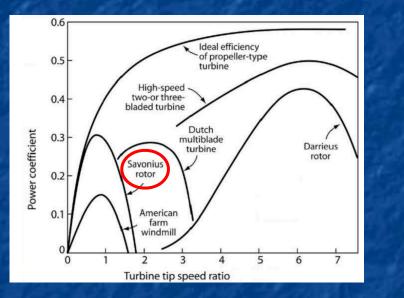
The Darrieus vs. Danish wind turbine size choice would (at best!) then be:

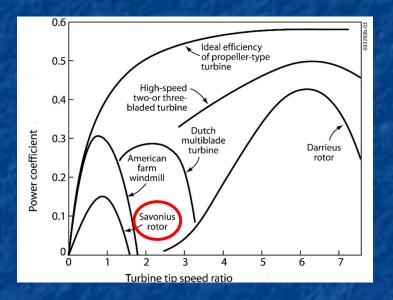


Today's Savonius turbines are almost at their much lower theoretical limit

VAWT Data Corruption:

Above are theory & data indicating Savonius Turbines are limited to ~15% efficiency But I then noted plots all over the Internet displaying a 30% efficiency peak Most cited absolutely no data source, as in the example at left below: 1





Then I spotted the "same" figure in a VAWT review I had used (shown above right) ² But look more closely at which curve it labels as the Savonius data! (Modern Danish turbines are best described by the "High-speed two-or three bladed turbine" curves)

1) http://www.icrepq.com/icrepq%2715/389-15-damota.pdf2) https://www.researchgate.net/publication/ 2) https://www.researchgate.net/publication/319678764_Review_Paper_Overview_of_the_Vertical_Axis_Wind_Turbines

Anyone can make a working wind turbine

The real problem is KEEPING it working!

"Anyone can make a working wind turbine The real problem is KEEPING it working!"

That is the overriding message of Paul Gipe's book about wind energy's origins: Wind Energy for the Rest of Us¹

The quote is my restatement of what has been called the **Danish Experience** Which was the school-of-hard-knocks collision of laboratory aerodynamics with the cold, wet, and often catastrophically windy reality of trying to get an actual turbine to produce affordable power for 20+ years

It's thus an essential addendum to my previous sections on aerodynamics!

1) Wind Energy for the Rest of Us, Paul Gipe, Windowrks.org (2016)

The very rough road to present day wind power: As told by Gipe, wind power's development has been anything but smooth He describes it as consisting of **two big misadventures + one final succes**s With the latter ultimately attributed to penny-pinching Danish farmers whose purchases led a small local farm machinery company to grow into today's largest worldwide producer of wind turbines The first misadventure was of government/corporate alliances in the U.S. & Germany Which were stimulated by the middle east oil embargos of the 1970's & 1980's In the U.S., the players were NASA and the Department of Energy teamed with corporate giants such as Boeing, General Electric and Westinghouse, along with utility companies such as California's Pacific Gas & Electric (PG&E)

The rationale behind such collaborations was clear: These agencies were our governmental bastions of aeronautics & power expertise These corporations mirrored that expertise, adding

manufacturing, sales & operational experience

This collaboration built BIG wind turbines Having sizes modern commercial wind power has only recently matched - decades later!

This is Boeing's 1987 "Mod-5b" with its 98 meter diameter rotor



Photo: http://www.windsofchange.dk/WOC-usastat.php

Those aerodynamic test-beds produced valuable lessons: Unfortunately, the paramount lesson was that they were very unreliable For instance, one of the more successful project prototypes (the "Mod 2") operated for a total of only 8658 hours (equivalent to ~ one year), it was available for operation only 37% of the time, and after being dynamited to the ground it was sold for scrap by PG&E

Excerpting some of Gipe's data from this NASA / DOE / industry collaboration:

	Early U.S. NASA/DOE Funded Experimental Turbines				
Turbine	Installed	Diameter	Swept Area	Capacity	Operating
		(meters)	(sq. meters)	(kW)	Hours
Mod 0A	1977	38	1141	200	13045
Mod 1	1979	61	2922	2000	0
Mod 2	1982	91	6504	2500	8658
Mod 5b	1987	98	7466	3200	20561

Excerpted from: Table 4-1, p. 54, Wind Energy for the Rest of Us, Paul Gipe, Windowrks.org (2016)

Why did these turbines have such short operational lifetimes? Real world wear and tear (especially in high winds) was the direct culprit But Gipe attributes the ultimate fault to these company's aviation culture: In commercial aviation, airplanes spend a LOT of time out of service on the ground being maintained, inspected and upgraded Planes can thus spend less than half of their time actually flying passengers With such a maintenance-intensive and downtime-accepting culture aerospace companies designed comparably maintenance hungry wind turbines Those turbines had downtimes often exceeding 60% ("Mod 2" above) And, outside in severe weather and thus hard to maintain, had brief lifetimes The bottom line: NASA & DOE terminated these R&D programs and none of the companies pursued further commercial development

The second misadventure: "The California Wind Rush" 1 This was another byproduct of the middle east oil embargos, when the state of California added its own green energy tax rebates to similar tax rebates being offered by the federal government California also first assessed its statewide wind resources, identifying three mountain passes as prime locations: Good Good Altamont, Tehachapi & San Gorgonio Passes 2 Altamo The state's Public Utilities Commission then encouraged Pache Pass California power companies to foster green energy They were then discovered to be colluding against it ¹ So the PUC then **ordered** those companies to begin adding their **own** generous incentives

> 1) Page 68-72, Wind Energy for the Rest of Us, Paul Gipe, Windowrks.org (2016) 2) https://en.wikipedia.org/wiki/Wind power in California FIGURE: http://zebu.uoregon.edu/2001/ph162/wreg.html



This led to a predictably California-style entrepreneurial stampede: ¹ Where instead of a few huge old companies developing a few huge turbines,

a whole array of small new companies developed a whole array of small turbines:







DAF



Enertech



1) Here I speak as someone who spent the first third of my life – and received all of my formal education - in California Photos from Paul Gipe's "Winds of Change" listing of early U.S. turbines: http://www.windsofchange.dk/WOC-usaturb.php

Many of these "California" turbines shared common characteristics: 1 Compared to turbines then being developed in Denmark, the "California" turbines were not only smaller but also much more lightly built But despite their smaller & lighter construction many actually used higher Tip Speed Ratios (i.e., higher blade speeds) which subjected those **lighter** structures to **larger** mechanical stresses Further, to take advantage of notoriously ephemeral government tax rebates, these turbines were often developed very quickly, with very little testing And because those subsidies rewarded units with higher stated power capacities, there was a severe temptation to strongly overstate those capacities THE RESULT? Once again many turbines fell well short of expectations in terms of both real power produced AND operational lifetimes 1) Page 68-72, Wind Energy for the Rest of Us, Paul Gipe, Windowrks.org (2016)

Bringing us to the effort that has defined modern Wind Power: Which took place largely on the weather-beaten farms of Denmark There, wind has traditionally been used to pump water & grind grain: But then, in the same post-middle-east-oil-embargo time frame, local tinkerers, inventors & academics began work on modernizing that tradition Consistent with that tradition, their would-be customers were not government agencies or public utilities, but farmers on isolated farms But rather than having "deep pockets," farmers are notoriously (and proudly) cheap And already risking their economic survival with the planting of each year's crops, they weren't going to increase risk by investing in unproven new technology The key word in that last sentence was "unproven"

Danish farmer's insisted on proof!

Thus, while Denmark had an "entrepreneurial stampede" resembling California's, entrepreneur's claims were not so eagerly accepted, nor was disappointing early turbine performance so easily tolerated Farmers instead pushed back, insisting on standardized up front testing, with the results then being openly disseminated, along with information on operational lifetimes & equipment failures via the newly formed **Danish Wind Turbine Owner's Association** This compelled (often painful) disclosure forced inventors & entrepreneurs: To address failures such as those encountered with new custom blade designs, often replacing custom parts with **standardized designs** then shared, and further refined, across Denmark's emerging Wind Power industry And to provide much improved wind turbine **over-speed protection measures**

But the farmer's skepticism then produced one final ironic twist: Farmer's were used to buying equipment from local farm equipment manufacturers, which, to survive, had carefully developed and protected their reputations for selling exceptionally cost-effective & reliable equipment As testing & disclosure drove entrepreneurs toward the "Danish" wind turbine design, what should then be more natural than for farm equipment manufacturers to begin manufacturing, distributing and maintaining that design? A local farm equipment company named Vestas took up that challenge And shortening the long story¹ of the "Danish Wind Turbine's" full development: Vestas is now WORLD'S LARGEST producer of commercial wind turbines

1) Page 58-68 and 75-81, Wind Energy for the Rest of Us, Paul Gipe, Windowrks.org (2016)

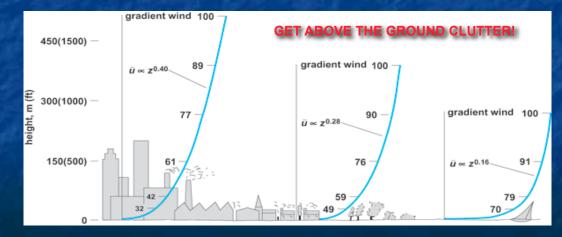
Trends in Commercial Wind Power

Wind Power is Changing

That is a point often underappreciated outside the wind power industry E.G., in **Wind Power II** (<u>pptx</u> / <u>pdf</u> / <u>key</u>) I'll review influential studies on Wind Power's: Economics (i.e., its LCOE = Levelized Cost of Energy) Its Energy Returned on Invested energy (EROI) Its toll upon birds and bats Those studies, done by serious economists, ecologists and social scientists, tend to base conclusions on averages of all existing Wind Power installations But existing installations span many, many technology generations And I will show how re-analyzing those data in terms of technology generation has sometimes led me to some very different conclusions (some more favorable, at least one less favorable) An Introduction to Sustainable Energy Systems: WeCanFigureThisOut.org/ENERGY/Energy home.htm

So where have we been, and where are we going with Wind Power? The recent past has been Danish, the foreseeable future will be Danish Understanding why has been perhaps my major goal in writing this note set In technical/textbook literature "turbine" is now synonymous with "Danish turbine:" A 3-bladed turbine, mounted on a wind-facing, tower-mounted, rotatable nacelle Evolution of that design is now almost entirely driven by these earlier figures

With their messages about reaching higher:

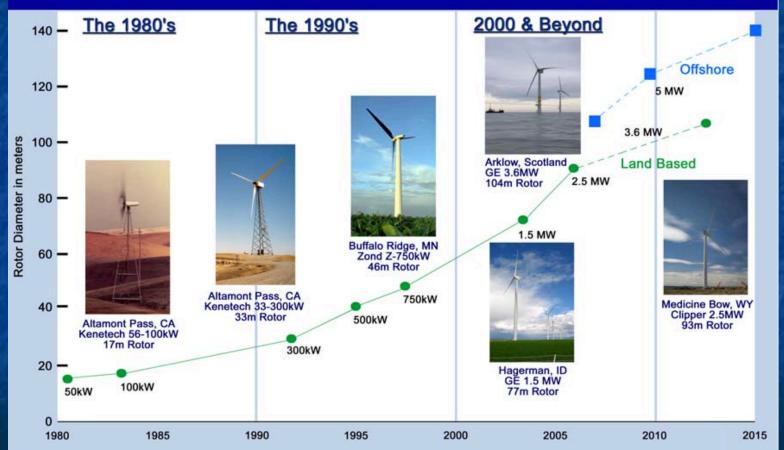






Turbines have thus been getting steadily bigger:

A U.S. National Renewable Energy Labs (NREL) plot of early U.S. turbines 1

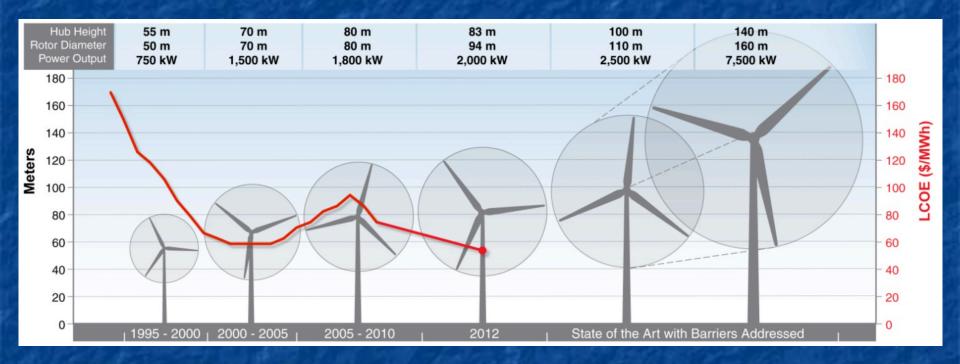


Evolution of U.S. Commercial Wind Technology

1) https://www.nrel.gov/docs/fy08osti/43374.pdf

And bigger:

A U.S. National Renewable Energy Labs (NREL) plot of more recent turbines 1



1) p. 5: https://www.nrel.gov/docs/fy14osti/61063.pdf

Assembly of these huge turbines is a "monumental" challenge:

Here: Installation of the nacelle on a Siemens 6 MW turbine, with the blades of its 154 meter diameter rotor waiting on the ground



https://www.siemens.com/press/en/presspicture/?press=/en/presspicture/2014/windpower-renewables/pn201403.php

As is just getting components TO the assembly site:



Figure: p. 81, Wind Vision Report – US DOE: https://www.energy.gov/eere/wind/wind-vision

The growing dominance of these larger turbines is shown here: In this Greentech Media figure showing how previously "BIG" 2 MW turbines ¹ are now being displaced by at least 4 MW turbines ¹

(which are now being displaced by 6 MW turbines ¹)

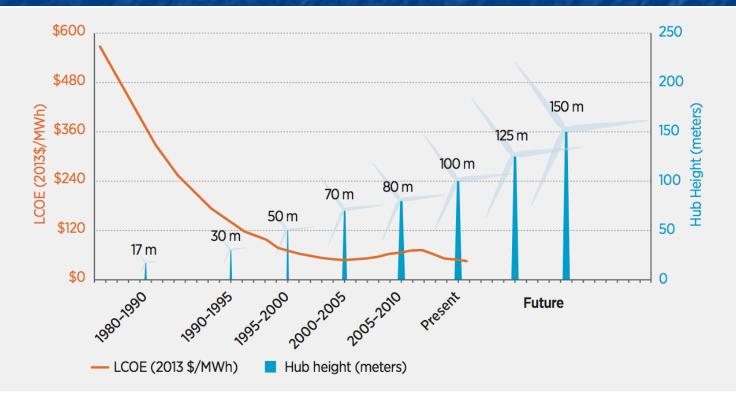


https://www.greentechmedia.com/ articles/read/an-illustrated-guide-tothe-growing-size-of-windturbines#gs.MmESzMk

1) Such "nameplate" power ratings ≠ Average power from such a turbine! They're the highest power that turbine could ever produce = It's CAPACITY

But physical growth has driven Wind Power prices steadily downward

As correlated here in the left axis plot of wind's "levelized" cost of energy (LCOE):



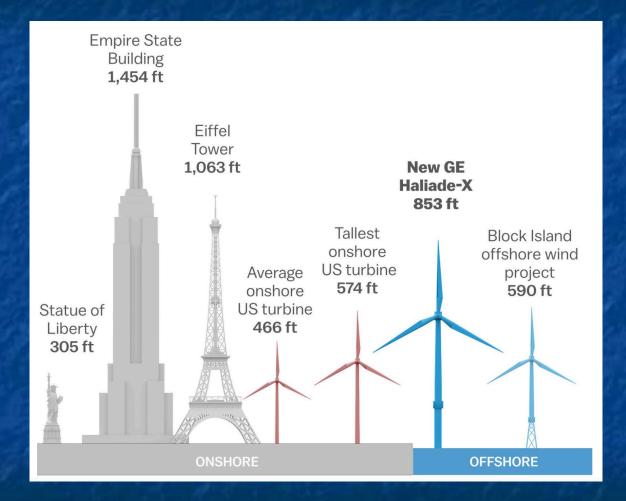
Note: LCOE is estimated in good to excellent wind resource sites (typically those with average wind speeds of 7.5 m/s or higher), excluding the federal production tax credit. Hub heights reflect typical turbine model size for the time period.

Source: Wiser and Bolinger [6]

Figure: p 63. Wind Vision Report – US DOE: https://www.energy.gov/eere/wind/wind-vision

Much larger turbines are already in the design stages:

Including GE's 12 MW wind turbine "expected to ship in 2020" 1



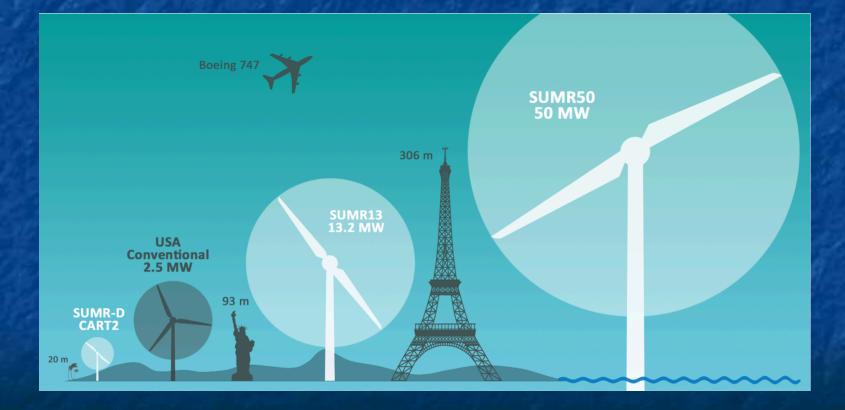
853 feet => 260 meters in height (with a 220 meter diameter rotor)

1) https://www.vox.com/energy-and-environment/2018/3/8/17084158/wind-turbine-power-energy-blades

And even larger turbines are being contemplated:

From Scientific American's 2017 article:

"World's Largest Wind Turbine Would Be Taller Than the Empire State Building Massive, flexible blades would bend with storm winds like the palm trees that inspired them"



1) https://www.scientificamerican.com/article/world-rsquo-s-largest-wind-turbine-would-be-taller-than-the-empire-state-building/

Those figures raise obvious questions about:

Integrating Wind Power into our **Economy**

Integrating Wind Power into our Grid

Integrating Wind Power into our Lives

Integrating Wind Power into our **Biosphere**

These and other broader challenges and possible impacts will be explored in my second web note set about Wind Power

There I will also explore the technology & impacts of **Offshore Wind Power**:

Wind Power II (<u>pptx</u> / <u>pdf</u> / <u>key</u>)

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